UNLV
UNIVERSITY OF NEVADA LAS VEGAS

Math 181
Recitation Quiz 4 (2.5 – 2.6, 2.8)
SOLUTIONS

Please notify me if you feel there is a typo in need of correction.
Typo fixed 10/14/09

1. Find when the function $f(x) = 6\sin(\pi x)$ has a horizontal tangent on $[-2\pi, 2\pi]$.

   SOLN: $[-2\pi, 2\pi] \approx [-6.28, 6.28]$

   $$f'(x) = 6\pi\cos(\pi x) \equiv 0 \text{ when } \pi x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}...$$

   $$x = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{7}{2}...$$

   So $x =$ $x = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{7}{2}, \pm\frac{9}{2}, \pm\frac{11}{2}$

2. Find when the second derivative is zero for the function $f(x) = 2x^4 - 3x^3 + 5x$.

   SOLN: $f'(x) = 8x^3 - 9x^2 + 5 \Rightarrow f''(x) = 24x^2 - 18x$

   This happens when $24x^2 - 18x = 6x(4x-3) \Rightarrow x = 0, \frac{3}{4}$

3. Differentiate $f(x) = \sqrt{\dfrac{\sin(\cos(x)) + 1}{e^{-2x+1} + x}}$

   SOLN:

   $$f'(x) = \frac{1}{2}\left(\frac{\sin(\cos(x)) + 1}{e^{-2x+1} + x}\right)^{-1/2} \frac{(e^{-2x+1} + x)[\cos(\cos x)\cdot\sin x] - [\sin(\cos(x)) + 1](e^{-2x+1}\cdot(-2) + 1)}{(e^{-2x+1} + x)^2}$$

4. Differentiate $f(x) = \sin^2(\pi x) + \sqrt{\sqrt{\sqrt{\pi}}}$.

   SOLN: $f'(x) = 2\sin^1(\pi x)\cdot\cos(\pi x)\cdot\pi + 0$

5. Differentiate $f(x) = x\cdot 3^x \cdot e^{\sin(x^2+2x+1)}$

   SOLN: $f'(x) = (x\cdot 3^x)\cdot(e^{\sin(x^2+2x+1)})' + (e^{\sin(x^2+2x+1)})\cdot(x\cdot 3^x)'$

   $$= (x\cdot 3^x)\cdot e^{\sin(x^2+2x+1)}(\sin(x^2+2x+1))' + (e^{\sin(x^2+2x+1)})\cdot(x\cdot(3^x)' + 3^x\cdot 1)$$

   $$= (x\cdot 3^x)\cdot e^{\sin(x^2+2x+1)}(\cos(x^2+2x+1))(2x+2) + (e^{\sin(x^2+2x+1)})\cdot(x\cdot(3^x\cdot\ln 3) + 3^x\cdot 1)$$

6. Find when the function $f(x)=\dfrac{x^3-2x^2+5x}{\lambda}$ has a horizontal tangent.

SOLN: $f'(x)=\dfrac{1}{\lambda}(x^3-2x^2+5x)'=\dfrac{1}{\lambda}(3x^2-4x+5)$

This has a root at $\dfrac{+4\pm\sqrt{16-4(3)(5)}}{2(3)}=\dfrac{+4\pm\sqrt{-44}}{6}$. No root, no horizontal tangents.

7. For $x^2+2xy-y^2+x=2$, use implicit differentiation to find *dy*/*dx*, and then find the equation of the tangent line to the curve at the point (1,2).

SOLN:

$$2x+2xy'+2y-2yy'+1=0$$

$$y'=\frac{-2x-2y-1}{2x-2y}$$

At the point (1,2) we have $y'(1,2)=\dfrac{-2-4-1}{2-4}=\dfrac{7}{2}$

8. (15 points) Find y' and y'' by implicit differentiation for $x^3+y^3=1$ (your answer should be in terms of *x* and *y*).

SOLN: $3x^2+3y^2y'=0 \Rightarrow y'=\dfrac{-x^2}{y^2}$

$$y''=\left(\frac{-x^2}{y^2}\right)'=\frac{y^2(-2x)+x^2(2yy')}{y^4}=\frac{y^2(-2x)+x^2(2y\frac{-x^2}{y^2})}{y^4}=\frac{-2xy^2-2\frac{x^4}{y}}{y^4}=\frac{-2xy^3-2x^4}{y^5}$$

9. (15 points) Find the equation of the tangent line, *y*(*x*), for $f(x)=x^{3/5}$ at *x* = 32. Estimate *f*(32.01) with this equation. What is the percentage error compared to what it should be?

SOLN: $f'(x)=\dfrac{3}{5}x^{-2/5}$

$f'(32)=\dfrac{3}{5}(32)^{-2/5}=\dfrac{3}{20}$ and $f(32)=8$

So $y(x)-8=\dfrac{3}{20}(x-32)\Rightarrow y(x)=\dfrac{3}{20}x-\dfrac{3}{20}\cdot 32+8=\dfrac{3}{20}x+\dfrac{16}{5}$

$$y(32.01)=\frac{3}{20}\left(\frac{3201}{100}\right)-\frac{16}{5}=8.0015$$

$$f(32.01)\approx 8.0014999$$

$$error=\left|\frac{8.0015-8.0014999}{8.0014999}\right|\approx 0.000001249\%$$

Note, students may be ‘off’ depending on the technology and precision they used.