UNIVERSITY OF NEVADA LAS VEGAS

Math 181
Recitation Quiz 3 (2.1 – 2.4)
SOLUTIONS

Open notes, open book. No more than 3 per group.
All questions worth 10 points. No work = No credit.

1. What is the definition of the derivative of a function *y* at *x* (use the format given in Definition 4 in section 2.1)?

   SOLN: $\lim_{h\to 0}\frac{y(x+h)-y(x)}{h}$

   This must match exactly, in particular it must have a *y*, and an *x*.

2. Using the definition in question 1, find the **slope** of the tangent line to the curve at any point for the function $f(x)=\frac{x-5}{x+2}$.

   SOLN:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{x+h-5}{x+h+2}-\frac{x-5}{x+2}}{h}=\lim_{h\to 0}\frac{1}{h}\left(\frac{x+h-5}{x+h+2}-\frac{x-5}{x+2}\right)$$

$$=\lim_{h\to 0}\frac{1}{h}\left(\frac{(x+h-5)(x+2)-(x-5)(x+h+2)}{(x+h+2)(x+2)}\right)$$

$$=\lim_{h\to 0}\frac{1}{h}\left(\frac{(x-5)(x+2)+h(x+2)-(x-5)(x+2)-h(x-5)}{(x+h+2)(x+2)}\right)$$

$$=\lim_{h\to 0}\frac{1}{h}\left(\frac{h(x+2)-h(x-5)}{(x+h+2)(x+2)}\right)=\lim_{h\to 0}\left(\frac{8}{(x+h+2)(x+2)}\right)=\frac{7}{(x+2)^2}$$

3. Using the definition in question 1, find the **equation** of the tangent line to the curve at the point $x = 0$ for the function $f(x)=x^2+2x$.

   SOLN:

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2+2(x+h)-(x^2+2x)}{h}$$

$$=\lim_{h\to 0}\frac{(x^2+2xh+h^2)+2x+2h-(x^2+2x)}{h}$$

$$=\lim_{h\to 0}\frac{h^2+2xh+2h}{h}=\lim_{h\to 0}\frac{2h+2xh+h^2}{h}=\lim_{h\to 0}(2+2x+h)=2+2x$$

So $f'(0)=2$

-or-

$$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h^2+2h}{h}=\lim_{h\to 0}(h+2)=2$$

So the equation of the tangent line through (0,0) is $y = 2x$

4. Find the derivative of the function $f(x)=\sqrt{x}$ using the definition from question 6. State the domain of the function and the domain of the derivative.

SOLN:

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$$
$$=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$
$$=\lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})}$$
$$=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

5. Prove (using the definition in question 6) that the derivative of a constant is zero.

SOLN: $\lim_{h\to 0}\frac{y(x+h)-y(x)}{h}=\lim_{h\to 0}\frac{c-c}{h}=0$

6. Prove (using the definition in question 1) that the derivative of a linear function is its' slope.

SOLN: $\lim_{h\to 0}\frac{y(x+h)-y(x)}{h}=\lim_{h\to 0}\frac{m(x+h)+b-(mx+b)}{h}=\lim_{h\to 0}\frac{mh}{h}=m$

7. Differentiate $f(x)=\sqrt{x}(x^2-1)$ by

a. Multiplying through and using the power rule

SOLN: $f(x)=x^{1/2}(x^2-1)=x^{5/2}-x^{1/2}\Rightarrow f'(x)=\frac{5}{2}x^{3/2}-\frac{1}{2}x^{-1/2}$

b. Using the product rule

SOLN: $f'(x)=\sqrt{x}(2x)+(x^2-1)\frac{1}{2}x^{-1/2}$

c. Simplify to show they are equivalent.

SOLN: $f'(x)=\sqrt{x}(2x)+(x^2-1)\frac{1}{2}x^{-1/2}=2x^{3/2}+\frac{1}{2}x^{3/2}-\frac{1}{2}x^{-1/2}=\frac{5}{2}x^{3/2}-\frac{1}{2}x^{-1/2}$

8. Differentiate $f(x)=\frac{x^2-4\sqrt[3]{x}}{x}$ by

a. Dividing through and using the power rule

SOLN: $f'(x)=(x-4x^{-2/3})'=1-4(-2/3)x^{-5/3}=1+\frac{8}{3}x^{-5/3}$

b. Using the quotient rule

SOLN: $f'(x)=\frac{x(x^2-4\sqrt[3]{x})'-(x^2-4\sqrt[3]{x})(1)}{x^2}=\frac{x\left(2x-\frac{4}{3}x^{-2/3}\right)-x^2+4\sqrt[3]{x}}{x^2}$

c. Simplify to show they are equivalent.

SOLN: $f'(x)=\frac{2x^2-\frac{4}{3}x^{1/3}-x^2+4\sqrt[3]{x}}{x^2}=\frac{x^2+\frac{8}{3}x^{1/3}}{x^2}=1+\frac{8}{3}x^{-5/3}$