Math 181
Recitation Quiz 2 (1.3 – 1.6)
SOLUTIONS

Please notify me if you feel there is a typo in need of correction.

1. Find $\lim_{x\to -3} \frac{x^2-2x-15}{x+3}$

$$\lim_{x\to -3} \frac{x^2-2x-15}{x+3} = \lim_{x\to -3} \frac{(x-5)(x+3)}{x+3} = \lim_{x\to -3} x-5 = -8$$

2. Find $\lim_{x\to 0} \frac{\sqrt{x+36}-6}{x}$

$$\lim_{x\to 0} \frac{\sqrt{x+36}-6}{x} \cdot \frac{\sqrt{x+36}+6}{\sqrt{x+36}+6} = \lim_{x\to 0} \frac{x+36-36}{x(\sqrt{x+36}+6)} = \lim_{x\to 0} \frac{1}{\sqrt{x+36}+6} = \frac{1}{12}$$

3. Find $\lim_{x\to -6^+} \frac{x^2+4x-12}{|x+6|}$

$$\lim_{x\to -6^+} \frac{(x+6)(x-2)}{x+6} = \lim_{x\to -6^+} x-2 = -8$$

4. Find $\lim_{x\to -6^-} \frac{x^2+4x-12}{|x+6|}$

$$\lim_{x\to -6^-} \frac{(x+6)(x-2)}{-(x+6)} = \lim_{x\to -6^+} -(x-2) = 8$$

5. Using information from 3 and 4, find $\lim_{x\to -6} \frac{x^2+4x-12}{|x+6|}$

The limit from the left and right are not the same, so it does not exist.
NOTE: You must be sure to evaluate | x + 6 | properly. It varies depending on the problem.

6. Find $\lim_{x\to \infty} \frac{x^2+2x^3+7x}{3x-4x^3+17x^2}$

$\frac{x^2+2x^3+7x}{3x-4x^3+17x^2}$ behaves like $\frac{2x^3}{-4x^3} = \frac{-1}{2}$, so $\lim_{x\to \infty} \frac{x^2+2x^3+7x}{3x-4x^3+17x^2} = \frac{-1}{2}$

7. Find $\lim_{x\to\infty}\frac{x^2+2x^3+7x}{3x-4x^4+17x^2}$

$\frac{x^2+2x^3+7x}{3x-4x^4+17x^2}$ behaves like $\frac{2x^3}{-4x^4}=\frac{-1}{2x}$, so $\lim_{x\to\infty}\frac{x^2+2x^3+7x}{3x-4x^4+17x^2}=0$

8. Find $\lim_{x\to-\infty}\frac{x^2+2x^3+7x^5}{3x-4x^4+17x^2}$

$\frac{x^2+2x^3+7x^5}{3x-4x^4+17x^2}$ behaves like $\frac{7x^5}{-4x^4}=\frac{-7x}{4}$, so $\lim_{x\to-\infty}\frac{x^2+2x^3+7x^5}{3x-4x^4+17x^2}=+\infty$

9. Find all asymptotes (vertical and horizontal) for $f(x)=\frac{(x-4)(4+3x)(x+2)}{(x+2)(6-3x)(x+12)}$

The function is undefined when $(x+2)(6-3x)(x+12)=0 \Rightarrow x=-2,2,-12$

But notice that x = –2 is a root of the numerator. So this is not an asymptote.

The vertical asymptotes are at 2 and –12

To find the horizontal asymptotes, we take $\lim_{x\to\pm\infty}\frac{(x-4)(4+3x)(x+2)}{(x+2)(6-3x)(x+12)}$.

$\frac{(x-4)(4+3x)(x+2)}{(x+2)(6-3x)(x+12)}$ behaves like $\frac{(x)(3x)(x)}{(x)(-3x)(x)}=\frac{3x^3}{-3x^3}=-1$

So there is a horizontal asymptote at –1

10. Sketch one **function** that has all of the following. Be sure to label…

a) a removable discontinuity
b) an infinite discontinuity
c) a jump discontinuity
d) a horizontal asymptote
e) a vertical asymptote
f) at least one horizontal tangent

Answers will vary. However, in addition to having the elements listed, they must be on *one* function, and your function needs to satisfy the definition of a function (one input has only one output).