Math 181

Recitation Quiz 1 (1.1 – 1.2)

SOLUTIONS

Please notify me if you feel there is a typo in need of correction.

1. Find the domain of $f(x) = \dfrac{x^2 - 1}{x^2 + 2x + 1}$.

   It would be when everywhere except, $x^2 + 2x + 1 = 0 \Rightarrow (x+1)^2 = 0 \Rightarrow x = -1$

2. Determine if it has holes or asymptotes, if so, identify them.

   $$f(x) = \frac{x^2 - 1}{x^2 + 2x + 1} = \frac{(x-1)(x+1)}{(x+1)^2} = \frac{x-1}{x+1}$$

   $f(x)$ has an asymptote at $x = -1$

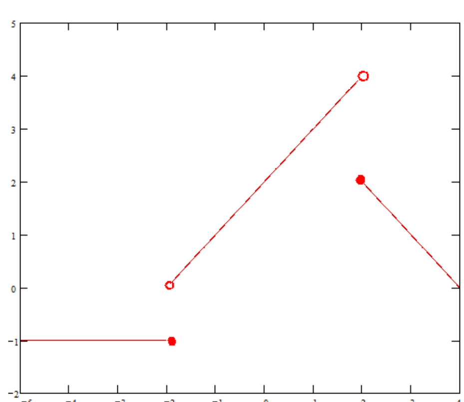
3. Evaluate and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ for $f(x) = \dfrac{x+1}{x-1}$.

   $$\frac{\dfrac{x+h+1}{x+h-1} - \dfrac{x+1}{x-1}}{h} = \frac{1}{h}\left[\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}\right] = \frac{1}{h}\left[\frac{(x+h+1)(x-1) - (x+1)(x+h-1)}{(x+h-1)(x-1)}\right]$$

   $$= \frac{1}{h}\left[\frac{(x+1)(x-1) + (h)(x-1) - (x+1)(x-1) - (h)(x+1)}{(x+h-1)(x-1)}\right]$$

   $$= \frac{1}{h}\left[\frac{(h)(x-1) - (h)(x+1)}{(x+h-1)(x-1)}\right] = \frac{(x-1) - (x+1)}{(x+h-1)(x-1)} = \frac{-2}{(x+h-1)(x-1)}$$

4. Sketch the graph of the function $f(x) = \begin{cases} -1 & x \le -2 \\ x+2 & |x| < 2 \\ 4-x & x \ge 2 \end{cases}$.



5. Determine whether the function $f(x)=\dfrac{x^3}{x^2+1}$ is odd, even or neither.

$$f(-x)=\frac{(-x)^3}{(-x)^2+1}=\frac{-x^3}{x^2+1}=-\frac{x^3}{x^2+1}=-f(x).$$ So it is odd.

6. For $f(x)=x^2-5,$ and $g(x)=\sqrt{(x-1)(x+1)}$, find and simplify $g\circ f(x)$.

$$g\circ f(x)=g\left[f(x)\right]=\sqrt{(x^2-5-1)(x^2-5+1)}$$
$$=\sqrt{(x^2-6)(x^2-4)}=\sqrt{(x+\sqrt{6})(x-\sqrt{6})(x+2)(x-2)}$$

7. For the question above, determine the domain of $g\circ f(x)$.

The domain of *f* is all real values.
The domain of *g* is $x\le -1$ or $x\ge 1$.

So the domain of the composition is the domain of *f* (all real) AND
$f\le -1$ or $f\ge 1$

$f\le -1\Rightarrow x^2-5\le -1\Rightarrow x^2-4\le 0\Rightarrow (x+2)(x-2)\le 0\Rightarrow x\in[-2,2]$

$f\ge 1\Rightarrow x^2-5\ge 1\Rightarrow x^2-6\ge 0\Rightarrow (x+\sqrt{6})(x-\sqrt{6})\ge 0\Rightarrow x\in(-\infty,-\sqrt{6}]\cup[\sqrt{6},\infty)$

So $x\in(-\infty,-\sqrt{6}]\cup[-2,2]\cup[\sqrt{6},\infty)$

Or you can see this with the sign diagram of the simplified result of the composition (what is on the inside of the square root):

$(x+\sqrt{6})$	− 0 + + + + + + + + +
$(x-\sqrt{6})$	− − − − − − − − − 0 + +
$(x+2)$	− − − 0 + + + + + + + +
$(x-2)$	− − − − − − − 0 + + + + +
	$-\sqrt{6}$ -2 $+2$ $\sqrt{6}$
	+ 0 − 0 + + + 0 − − 0 + +

The domain is $(-\infty,-\sqrt{6}]\cup[-2,2]\cup[\sqrt{6},\infty)$

8. When is the function $\dfrac{\sin(x-\pi)}{\cos\left(\dfrac{x}{2}+\pi\right)}$ undefined?

$\dfrac{\sin(x-\pi)}{\cos\left(\dfrac{x}{2}+\pi\right)}$ is undefined when $\cos\left(\dfrac{x}{2}+\pi\right)=0$. So,

$$\frac{x}{2}+\pi=\ldots\frac{-5\pi}{2},\frac{-3\pi}{2},\frac{-\pi}{2},\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}\ldots$$

$$x=\ldots 2\left(\frac{-5\pi}{2}-\pi\right),2\left(\frac{-3\pi}{2}-\pi\right),2\left(\frac{-\pi}{2}-\pi\right),2\left(\frac{\pi}{2}-\pi\right),2\left(\frac{3\pi}{2}-\pi\right),2\left(\frac{5\pi}{2}-\pi\right)\ldots$$

$$x=\ldots-7\pi,-5\pi,-3\pi,-\pi,\pi,3\pi\ldots$$

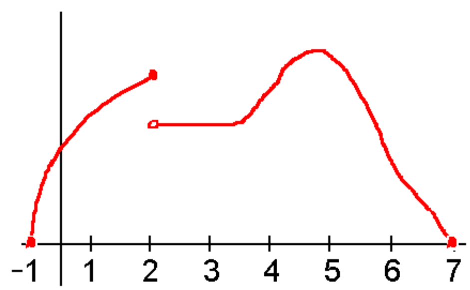
9. When is the function $\dfrac{\sin(x-\pi)}{\cos\left(\dfrac{x}{2}+\pi\right)}$ equal to zero?

The function is equal to zero when $\sin(x-\pi)=0$ (provided it exists).

$$x-\pi=\ldots-3\pi,-2\pi,-\pi,0,\pi,2\pi,3\pi\ldots$$

$$x=\ldots-2\pi,-\cancel{1\pi},0\pi,\cancel{1\pi},2\pi,\cancel{3\pi},4\pi\ldots=\ldots-2\pi,0,2\pi,4\pi\ldots$$

10. For the function pictured, identify (using interval notation) where it is increasing, decreasing and constant.



(–1,2) increasing
(2, *about* 3.8) constant
(*about* 3.8,5) increasing
(5, 7) decreasing

NOTE: all intervals are open.
The *about* values should match up, and don't have to be exact.
All should be OPEN intervals.