Section 5.5 – Substitution

A Look Back:

- Recall the chain rule, where for example $\frac{d}{dx}e^{x^2} = e^{x^2}(2x)$.
- Clearly, since we know integrals undo derivatives, we would have $\int e^{x^2}(2x)dx = e^{x^2} + c$

  If we let $u = x^2$, we would get $\frac{du}{dx} = 2x$, or $du = 2x\ dx$.

  Substituting u and du into the integral, we would have $\int e^u du = e^u + c = e^{x^2} + c$
- This 'method' is the chain rule used backwards.
- You CANNOT evaluate an integral of a product (like above) without using substitution.
- Formally, the definition of the substitution rule is $\int f(g(x))\ g'(x)\,dx = \int f(u)\,du$ where $u = g(x)$

Substitutions with Constants:

- If $g(x) = cx$, then setting $u = cx$ yields $\frac{du}{dx} = c$, or $dx = \frac{du}{c}$
- *Example. Evaluate* $\int e^{2x} dx$

Other Substitutions Require More Pieces:

- *Example. Evaluate* $\int \frac{x}{(x^2+1)^2} dx$

- *Example. Evaluate* $\int y^3\sqrt{2y^4-1}\,dy$

- *Example. Evaluate* $\int \frac{x^2}{\sqrt{1-x}}\,dx$

<u>*Definite Integrals:*</u>

- So long as you change back to *x*, these are no different than the above problems.
- *Example. Evaluate* $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$

Average Value:

- We define the average value of f on the interval $[a,b]$ as $\frac{1}{b-a}\int_a^b f(x)\, dx$.
- *Example. Find the average value of* $f(x) = e^{2x}$ *on [1,2]*