

Section 5.2 – The Definite Integral and 5.3 Evaluating Definite Integrals

Notation:

- Earlier we learned about antiderivatives. There are two ways of thinking about antiderivatives (also called integrals).

- There is a _____, which is the antiderivative over a specified range.

The antiderivative of $f(x)$ from a to b is written $\int_a^b f(x) dx$.

And if the antiderivative of $f(x)$ is given by $F(x)$, we can evaluate the above

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

The final result will always be a constant, and you need not worry about the arbitrary constant.

- There is an _____, which is not specified over any range

The antiderivative of $f(x)$ is written $\int f(x) dx$.

And if the antiderivative of $f(x)$ is given by $F(x)$, we can evaluate the above

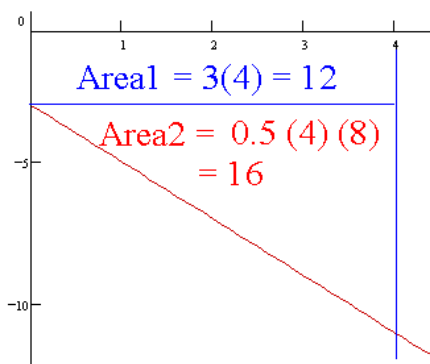
$$\int f(x) dx = F(x) + c$$

The final result will be a function, and will have the arbitrary constant added to the end.

- The concept of _____ relates to the definite integral. In particular, the definite integral will assign a negative sign to any area below the x axis, and a positive to any area above, and add them together for a total. If you want to find the _____, you will need to identify the roots of the interval and break the definite integral into different pieces.

Be Careful of the Sign:

- Let's look at the physical area of the function $f(x) = -2x - 3$ on $[0,4]$.



The total *physical area* is equal to $16 + 12 = 28$.

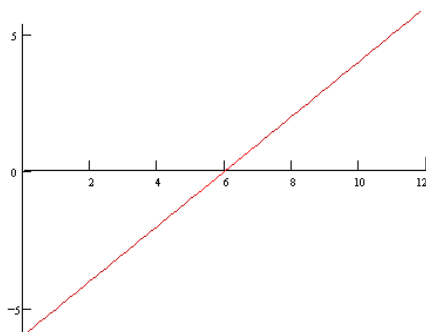
- We have seen that the integral is related to physical area, in that for $f(x)$ positive, $\int_a^b f(x) dx$ is the area between the curve and the x axis.

- *Let's evaluate the antiderivative of $f(x) = -2x - 3$.*

- This is **negative!** In fact, although the concepts are related they are not equivalent! Physical area is not concerned with the sign of the answer, whereas the integral is.

Two Related Problems:

- *Find the physical area between $f(x) = x - 6$ and the x axis on $[0, 12]$*



Down the Road:

- Eventually we will no longer use Reimann Sums and geometry to find physical area. But using integration to find physical area, you need to pay attention to the signs (above and below the x axis) and ‘break up’ the integral into parts.
- *Example. Find the physical area between the curve $f(x) = x^2 - 16$ on $[0, 8]$.*

