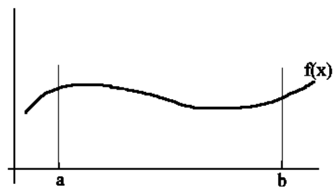
Section 5.1 – Areas and Distances

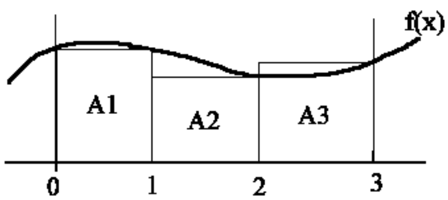
Reimann Integration

- We will learn later that for a function $f(x)$ that is positive and continuous on $[a,b]$ that

$$\int_a^b f(x)dx = \text{Area bound between } f\text{, the } x\text{-axis, } a \text{ and } b$$



- If we want to get an estimate for this without actually integrating, we use a technique call Reimann Integration.
- Let $f(x)$ be positive and continuous on [0,3] and fix three subintervals of equal length.



We can get an estimate by summing the areas of A1, A2 and A3

- We need to find the area of each box, which will just be lw, where

  w = the distance of each subinterval.

  In this case, since we picked $n = 3$, we have $w = \frac{b-a}{n} = \frac{3-0}{3} = 1$.

  l = the height of the box.

- As you can see from the picture, the height is the function value at the right* corner

  l for A1 is $f(1)$
  l for A2 is $f(2)$
  l for A3 is $f(3)$

* There are several types of Reimann Integrals, but we will focus on what is called a Right Sum, where we use the top RIGHT corner to give us our box.

- To simplify the way we write all this, we rename all the *x* values…

  $x_0 = a$
  $x_1 = 1$
  $x_2 = 2$
  $x_3 = 3$

  And call $w = \Delta x$

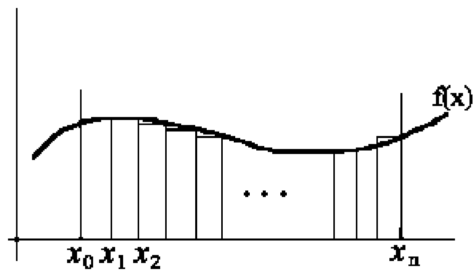
- Then $\text{A1} = \Delta x \cdot f(x_1)$

  $\text{A2} = \Delta x \cdot f(x_2)$

  $\text{A3} = \Delta x \cdot f(x_3)$

- This is how the method begins… But the more subintervals we take (i.e. the larger *n* is) the more accurate our estimate is. So we get rid of $n = 3$ and keep it arbitrary.

- Now we have $w = \Delta x = \dfrac{b-a}{n}$



- And our points are:

$$x_0 = a$$
$$x_1 = x_0 + \Delta x = a + \Delta x$$
$$x_2 = x_1 + \Delta x = a + 2\Delta x$$
$$x_3 = x_2 + \Delta x = a + 3\Delta x$$
$$\vdots$$
$$x_n = a + n\Delta x$$

- So the area of our "boxes" are estimated by:

$$\text{A1} = \Delta x \cdot f(x_1)$$
$$\text{A2} = \Delta x \cdot f(x_2)$$
$$\text{A3} = \Delta x \cdot f(x_3)$$
$$\vdots$$
$$\text{A}n = \Delta x \cdot f(x_n)$$

- Summing all these we get the TOTAL area, which is

$$\text{A1+A2+A3+}\ldots\text{+A}n = \sum_{i=1}^{n} \text{A}i = \sum_{i=1}^{n} \Delta x\, f(x_i)$$

- Now to finish, since we want *n* as large as possible, we take the limit as *n* tends to infinity.

- So, the ______________________________ is found by evaluating

Some Formulas You Need:

1. $\sum_{i=1}^{n} 1 = n$

2. $\sum_{i=1}^{n} c \cdot a_i = c \cdot \sum_{i=1}^{n} a_i$

3. $\sum_{i=1}^{n} a_i + b_i = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

4. $\sum_{i=1}^{n} c = cn$

5. $\sum_{i=1}^{\infty} i = \frac{n(n+1)}{2}$

6. $\sum_{i=1}^{\infty} i^2 = \frac{n(n+1)(2n+1)}{6}$

7. $\sum_{i=1}^{\infty} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

The Method for Right Reimann Integration:

1. $\Delta x = \frac{b-a}{n}$ Plug in *b* and *a*

2. $x_i = a + i \cdot \Delta x$ Plug in above value and *a*

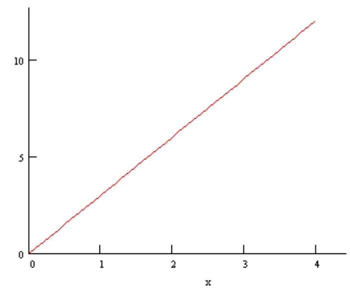
3. $\sum_{i=1}^{n} \Delta x \cdot f(x_i)$ Plug in above values and f

4. Simplify. Use formula(s) from above. You should be able to eliminate the sum symbol.

5. Take the limit, $\lim_{n \to \infty} \sum_{i=1}^{n} \Delta x \cdot f(x_i)$.

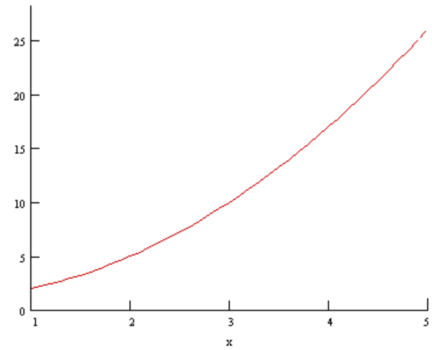
Example 1:

- Find area using the Right Reimann Integral for $f(x) = 3x$ on [0,4].



Example 2:

- Find area using the Right Reimann Integral for $f(x) = x^2 + 1$ on [1,5].



Note:

- Recall from last chapter, we found that the ____________________ of $f(x) = x^2 + 1$ is equal to $F(x) = \frac{x^3}{3} + x + c$.

  If we subtract $F(1)$ from $F(5)$ we get $F(5) - F(1) = \frac{5^3}{3} + 5 + c - \left(\frac{1^3}{3} + 1 + c\right) = \frac{136}{3}$!