

Section 4.7 – Antiderivatives

A Recap of Some Derivatives:

- Let's recall some derivatives we have seen so far...

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} e^{4x} = 4e^{4x}$$

$$\frac{d}{dx} \cos(2x^2 + x) = -(4x + 1) \sin(2x^2 + x)$$

- If we wanted to 'undo' the derivative, we could say

$$\text{Antiderivative}[\cos x] = \sin x$$

$$\text{Antiderivative}\left[\frac{1}{x}\right] = \ln x$$

$$\text{Antiderivative}[4e^{4x}] = e^{4x}$$

$$\text{Antiderivative}[-(4x + 1) \sin(2x^2 + x)] = \cos(2x^2 + x)$$

- In this section we will not focus much on *finding* this antiderivative, we will just be identifying the relationship between them.

The Arbitrary Constant:

- Look at the functions below with their derivatives, what do you notice?

$$\frac{d}{dx} (\sin x + 17) = \cos x$$

$$\frac{d}{dx} (\sin x - 15) = \cos x$$

$$\frac{d}{dx} (12 + \ln x) = \frac{1}{x}$$

$$\frac{d}{dx} (e + \ln x) = \frac{1}{x}$$

$$\frac{d}{dx} (e^{4x} - 7) = 4e^{4x}$$

$$\frac{d}{dx} (e^{4x} + 8) = 4e^{4x}$$

- Adding (or subtracting) a constant to a function only shifts it up or down. It does not affect the derivative (or shape) of that function.

- In that regard, any time you take the antiderivative of a function, you end up with what is called an arbitrary constant:

$$\text{Antiderivative}[\cos x] = \sin x + c$$

$$\text{Antiderivative}\left[\frac{1}{x}\right] = \ln x + c$$

$$\text{Antiderivative}[4e^{4x}] = e^{4x} + c$$

$$\text{Antiderivative}[-(4x+1)\sin(2x^2+x)] = \cos(2x^2+x) + c$$

The “Easy” Antiderivative:

- Recall $\frac{d}{dx}x^n = n \cdot x^{n-1}$
- Shifting the value of n by 1 we see $\frac{d}{dx}x^{n+1} = (n+1) \cdot x^n$
- Dividing by $n+1$ we find $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$

- So the antiderivative of a function to a power is given by

$$\text{Antiderivative}[x^n] = \frac{x^{n+1}}{n+1} + c$$

- *Example. Find the antiderivative of $f'(x) = 2x^2 - 3x + 7$ (find f)*

- *Example. Find the antiderivative of $f'(x) = 5x^{-3} + 8x^2 + 7x$ (find f)*

- Don't forget the arbitrary constant!
- **Q: What value of n will NOT work in the formula above? Have we seen it already?**
A: _____
