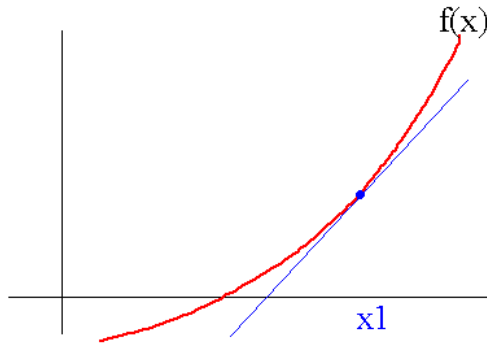


Section 4.6 – Newton’s Method

Deriving Newton’s Method:

- Let’s say we want to find the equation of the tangent to $f(x)$ at $x = x_1$.



Recall the slope of the tangent line at x_1 is $f'(x_1)$.

For the equation of the tangent through $(x_1, f(x_1))$, we have $y = f'(x_1)(x - x_1) + f(x_1)$.

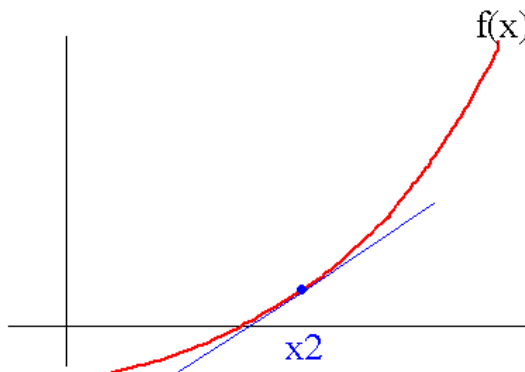
Now this line will have a root when $y = 0$, or

$$0 = x \cdot f'(x_1) - x_1 \cdot f'(x_1) + f(x_1)$$

$$x_1 \cdot f'(x_1) - f(x_1) = x \cdot f'(x_1)$$

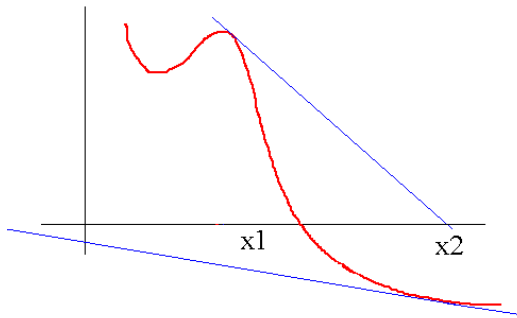
$$x = \frac{x_1 \cdot f'(x_1) - f(x_1)}{f'(x_1)} = x_1 - \frac{f(x_1)}{f'(x_1)} \quad \leftarrow \text{we call this } x_2$$

- If we continue in this fashion, we can use x_2 to find another root x_3 , where $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$



- The further we continue this, the more possible it is to get closer we get to the *actual* root of $f(x)$.
- Be careful** because this does not always work. You need to have a function whose derivative is not close to zero for the point in question. Concavity changes at or near the point could also cause problems. And it is imperative that you start with a “good” initial guess.

- Not starting with a good initial guess could lead to a problem like this one:

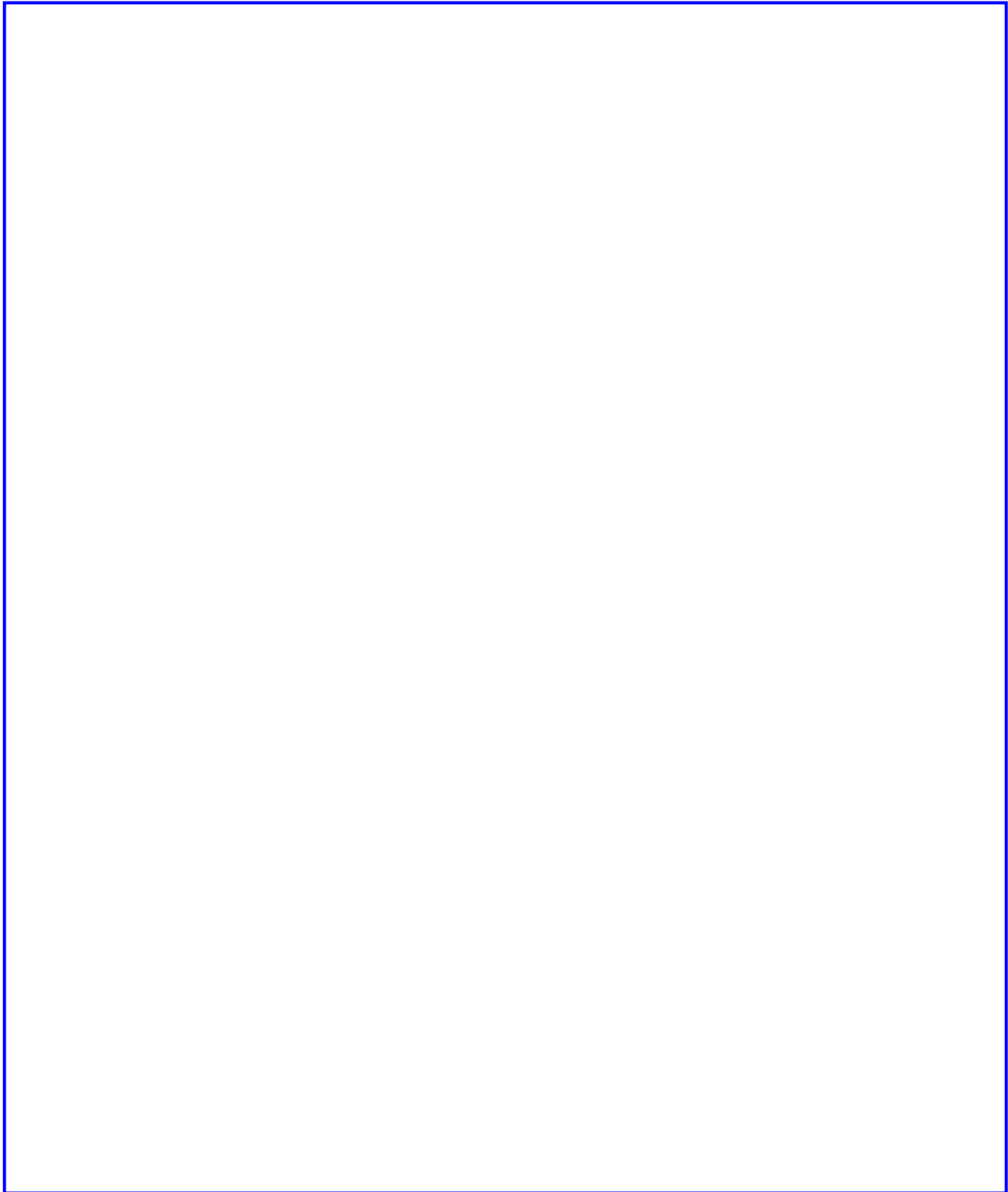


Newton's Method:

- You are usually provided with a function, and an initial guess, and sometimes a stopping criteria. If you are not provided an initial guess, this can be a challenging task that is crucial to the success of Newton's method, and you may want to graph the equation to determine the best initial guess.
- The method for function $f(x)$ and initial guess x_n :
 1. Find the derivative of the function $f'(x_n)$.
 2. Find $f(x_n)$ and $f'(x_n)$.
 3. Find $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.
 4. Find $f(x_{n+1})$ and see how close it is to zero.
 5. If it is close enough*, stop. If it is not, continue by finding $x_{n+2} = x_{n+1} - \frac{f(x_{n+1})}{f'(x_{n+1})}$.
 6. Repeat
- * 'Close enough' means different things to different people, if you are not provided with a stopping criteria, be sure to justify your answer.
- This is online as an Excel sheet where you can check your work for any polynomial up to degree five. Check it out!

Some Worked Problems:

- *Example. Use Newton's method to find x_2 and x_3 for $x^5 + 2 = 0$ with initial guess $x_1 = -1$.*



- *Example. Use Newton's method to find an approximate root for $x^3 - 3 = 0$ with initial guess $x_1 = 1$.*

