Section 4.3 – Derivatives and the Shapes of Graphs

- First of all, please note that the derivative of a function doesn't **affect** the shape of the graph, but **explains it**.

Increasing/Decreasing:

- We have already seen that when a function is increasing, its derivative is **positive**.
- And when a function is decreasing, its derivative is **negative**.

Local Max and Min:

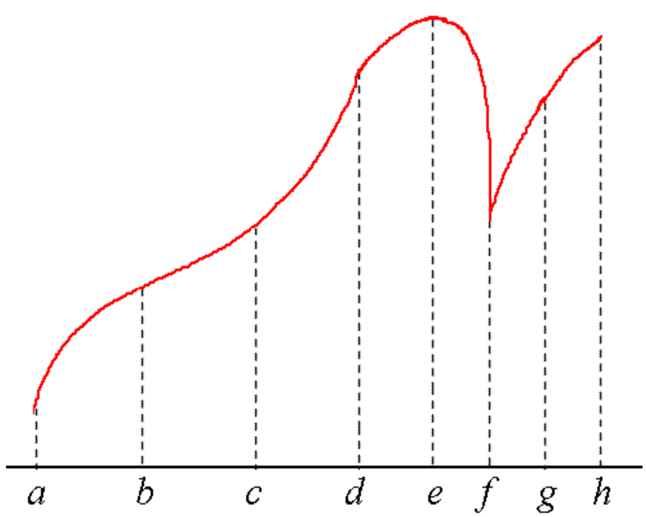
- If the derivative changes from positive to negative at a point c, then c is a **local maximum**.
- If the derivative changes from negative to positive at a point c, then c is a **local minimum**.
- If the derivative does not change sign at a point c, then it is **not a local max or min**.

Concavity:

- Recall when the second derivative of a function is positive, it is **concave up**. When the second derivative of a function is negative, it is **concave down**.
- Any point where the second derivative is zero is called a **point of inflection**, where the function changes from concave up to down (or down to up).

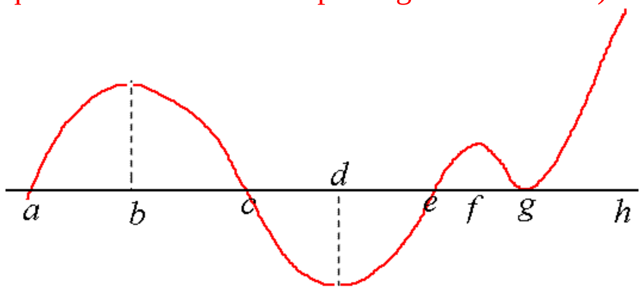
Examples:

- Q: For the graph below identify intervals/points where the function is increasing/decreasing, concave down/up, horizontal tangents, max, min, inflection points and critical points.



A:
The function is increasing on (a,e) and (f,h).
Decreasing on (e,f).
Horizontal tangent at e.
Global max at e.
Global min at a.
Local max at e and perhaps you could consider h one as well.
Local min at f and perhaps you could consider a one as well.
Inflection points at b, d
Concave up on (b,d)
Concave down on (a,b), (d,f) and (f,h)
Critical points at d, e and f.

- Q: Given the graph of the derivative (below) what can you say about the function itself (same questions as before except for global max/min)?



A:
The function is increasing when the derivative is positive, on (a,c), (e,g) and (g, h).
Decreasing when the derivative is negative, on (c,e).
Horizontal tangent when the derivative is zero, at a, c, e and g.
Local max when the derivative goes from positive, to zero, then negative, at c.
Local min when the derivative goes from negative, to zero to positive, at a and e.
Inflection point when the derivative has a horizontal tangent at b, d and g.
Concave up when the first derivative is increasing, on (a,b), (d,f) and (g,h).
Concave down when the first derivative is decreasing, on (b,d), (f,g).
Critical points when the derivative is 0 or undefined, at a, c, e and g.

- *Example. For $f(x) = x \ln x$, find when it is increasing/decreasing, max/min, concavity and points of inflection.*

The function is defined for $x > 0$, so this is our domain.

$$f'(x) = x\frac{1}{x} + \ln x$$
$$= 1 + \ln x$$

The first derivative is defined for $x > 0$.

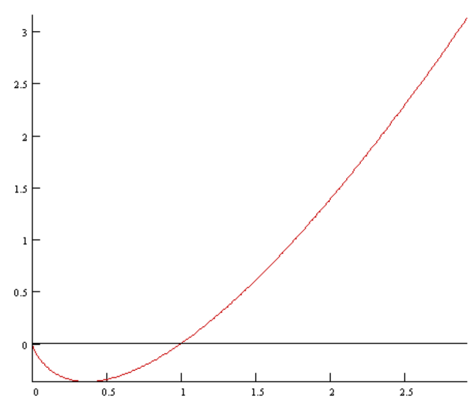
The first derivative is zero when:

$$1 + \ln x = 0$$
$$\ln x = -1,$$
$$x = e^{-1} \approx 0.37$$

$$f''(x) = \frac{1}{x}$$

Putting this information into a sign chart, we can see…

f''	+	+	+		+	+	+
f'	−	−	−	0	+	+	+
0				0.37			



The function is decreasing on (0,0.37), increasing (0.37,infinity).
There is a local min at 0.37
The function is always concave up, there are no inflection points.