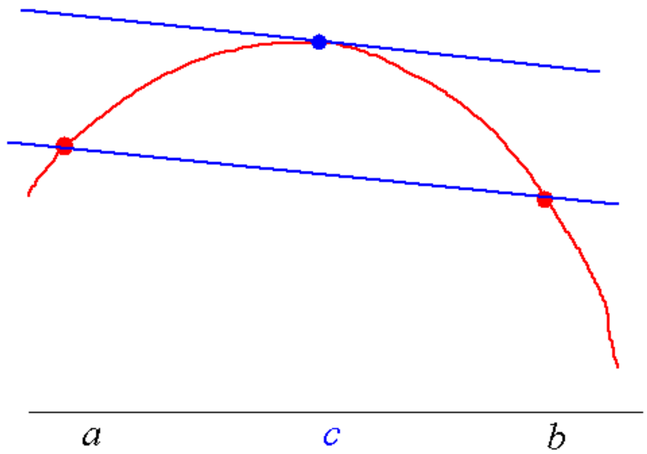
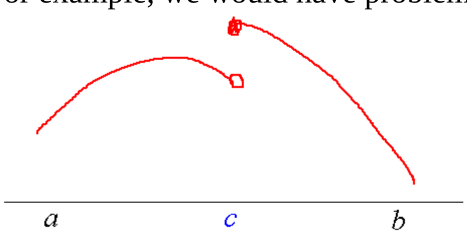
Section 4.2 – The Mean Value Theorem

Recall:

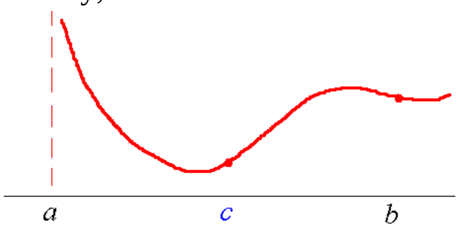
- When we first looked at the definition of the derivative, we started by looking at the slope of secant lines. Originally we picked the point *c* and another (which we called *c* + *h*).
- We could have also taken any two points surrounding *c*, and looked at the line connecting *a* and *b*. And then we could have moved this up until it was tangent to the curve at the point *c*.



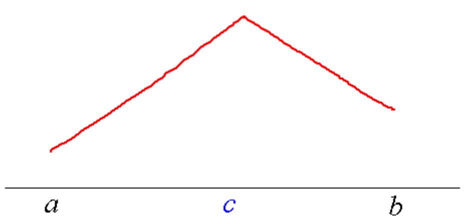
- Q: What conditions were important to us at that time?
  A: ____
- For example, we would have problems if we had a function that looked like:



Actually, it should be continuous at *a* and *b* as well. *f*(*a*) not defined below causes problems:



It should be differentiable on the interval (*a*,*b*), or we could run into a problem like:



where the derivative at *c* is not even defined.

- So to be able to do this in a general sense, we need to specify that *f* is ______________________ ________________________.
- Q: What would be the slope of this secant line if $f(a) = f(b)$?
  A: ____

The Theorems:

- _________________**:** Let f be a function that is continuous on $[a,b]$ and differentiable on (a,b). Then there is a c in (a,b) with $f'(c) = \frac{f(b) - f(a)}{b - a}$.
- _________________**:** Let f be a function that is continuous on $[a,b]$ and differentiable on (a,b) with $f(a) = f(b)$. Then there is a c in (a,b) with $f'(c) = 0$.

Examples:

- *Example. Verify* $f(x) = x\sqrt{x+6},\ [-6,0]$ *satisfies Rolles and find all such c's.*

- *Example. Let* $f(x)=(x-1)^{-2}$. *Show that f(0)=f(2) but there is no number c in (0,2) so* $f'(c)=0$. *Why not?*

- *Example. Verify that* $f(x) = \dfrac{x}{x+2}$ *satisfies the MVT on [1,4]. Find c.*