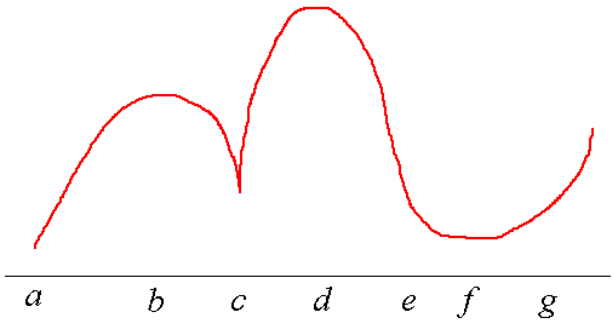
Section 4.1 – Max and Min Values

Horizontal Tangents:

- We have looked at graphs and identified _____, or places where the slope of the tangent line is zero.
- Q: For which *x* values does the following function have a horizontal tangent?



A: _____

- Take a look at each horizontal tangent again. Notice that to the left and to the right, the sign of the derivative is different.
- For example, to the left of point *b* the derivative is _____ (function is _____) and to the right of point *b* the derivative is _____ (function is _____).
- Also notice that when we say "to the left of *b*" and "to the right of *b*" we mean values that are really "close" to the value of *b*. Another point, say *a* or *c*, would be considered too "far" away.
- Also note that the point itself is a maximum or minimum compared to the values *near* it. This is called a _____ or _____ (sometimes called _____ or _____).
- Notice point *c*. It is not a horizontal tangent, but to the left of *c* the function is decreasing and to the right of *c* the function is increasing. It is also considered to be a relative min, but the derivative there is undefined (recall the derivative is undefined at sharp corners).

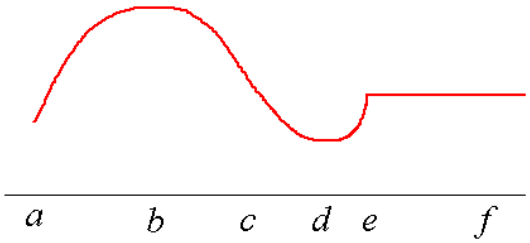
Critical Points:

- A **critical point** (sometimes called a **critical number**) of a function *f*(*x*) is when $f'(x) = 0$ or $f'(x)$ is undefined.

  Q: What are the critical points of the graph pictured above?
  A: _____

- Critical points include both horizontal tangents (derivative is zero), and places where the derivative is undefined (which could be corners, jumps or asymptotes). This does not mean the point is a maximum or minimum value. But if a point is a max or min, it will be a critical point.

- *Example. Identify the critical points, horizontal tangents, local max/min in the picture below.*

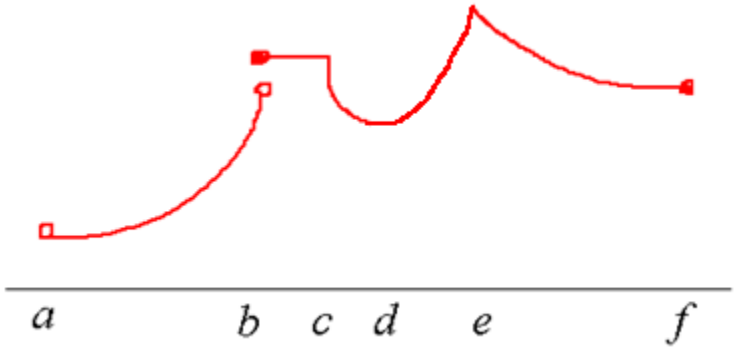


- So to recap -- relative max or min implies critical point, but critical point does not imply relative maximum or minimum.

Formal Definitions:

- A function f has a _____ at c if $f(c) \geq f(x)$ for all x near c.
- A function f has a _____ at c if $f(c) \leq f(x)$ for all x near c.
- Note that the equality allowed in the definition (according to our book) is what makes e in our example above a maximum value. Some people/books will not allow for equality in the definitions above, and use strict inequalities. For purposes of this class, we will stick with the books definition.
- A function f has an _____ (or _____) at c if $f(c) \geq f(x)$ for all x in the domain of f.
- A function f has an _____ (or _____) at c if $f(c) \leq f(x)$ for all x in the domain of f.
- Again, some people/books would use strict inequalities.
- Q: What is the difference between a global max and a local max?
  A: _____
  _____
- Q: Is it possible to have more than one global max (or min)?
  A: _____
  _____

- *Example: For each point in the picture below, is it a… local max, local min, global max, global min, critical point?*

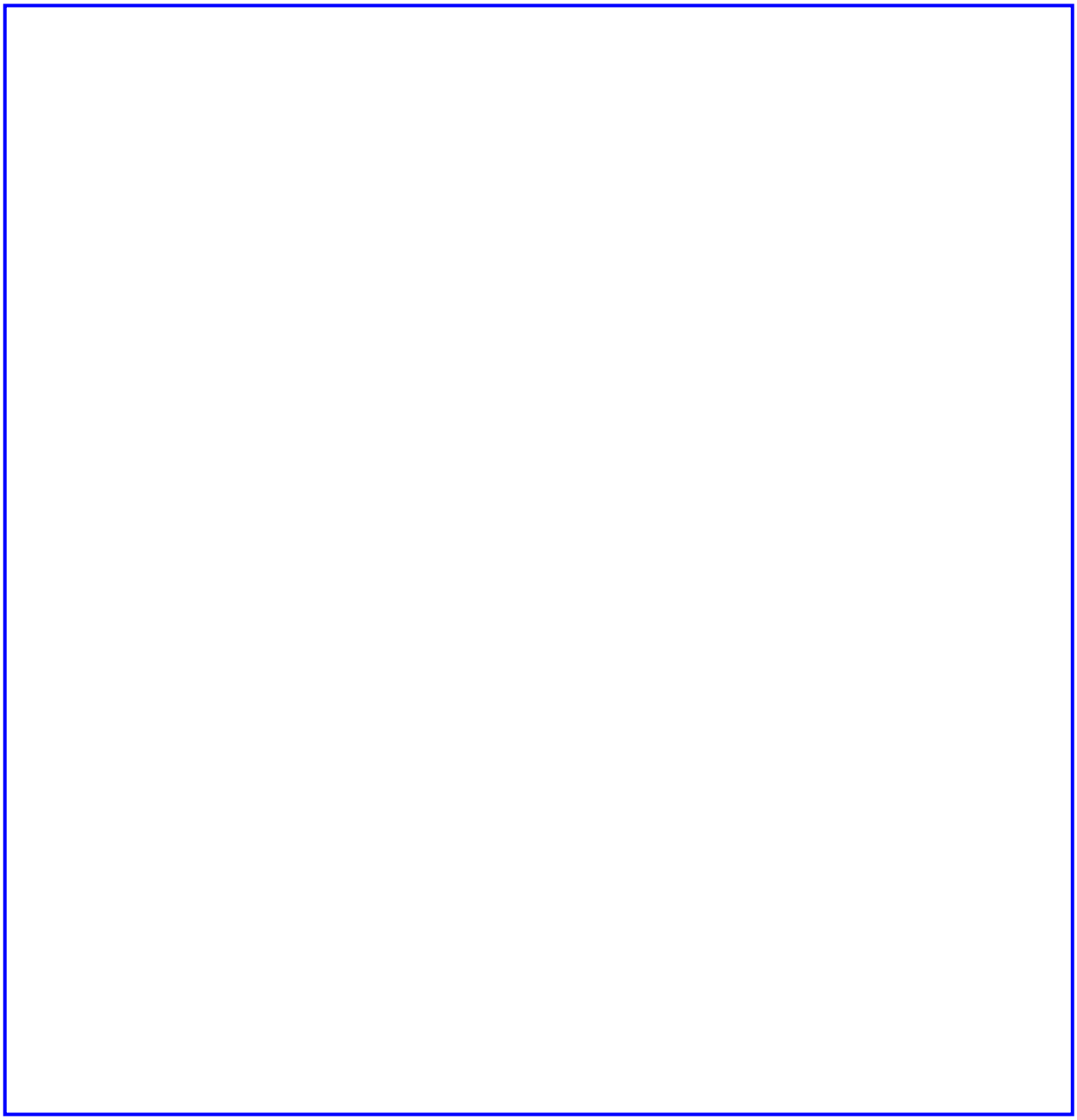


- _____ says that if f has a local max or min at c, and if $f'(c)$ exists, then $f'(c) = 0$.

How to Find and Identify Critical Points for an Equation:

- To find critical points from an equation, simply find the derivative. Anywhere the derivative is zero or undefined you have a critical point.
- If the critical point, c, comes from when the derivative is zero ($f'(c) = 0$), it could be…
  – A local max
  $f''(c) < 0$. Also, $f'(c^-) > 0$ and $f'(c^+) < 0$ (c^- is a value immediately to the left of c, c^+ is right).
  – A local min
  $f''(c) > 0$. Also, $f'(c^-) < 0$ and $f'(c^+) > 0$.
  – A global max on $[a, b]$
  $f''(c) < 0$, f is continuous on the interval $[a,b]$ and $f(c) \geq f(d) \quad \forall d \in [a,b]$.
  – A global min on $[a, b]$
  $f''(c) > 0$, f is continuous on the interval $[a,b]$ and $f(c) \leq f(d) \quad \forall d \in [a,b]$.
  – An inflection point
  $f''(c) = 0$.
- If the critical point comes from when the derivative is undefined, it could be…
  – A "sharp" corner.
  – A jump discontinuity.
  – An infinite discontinuity (asymptote).
  All of these will be identifiable in the original function.

- *Example. Find the cp's (and identify) for* $f(x) = \dfrac{x^2 - 4}{x^2 + 4}$ *on* [–4,4]



- Q: Why could there not be a value that exceeds 0.6 on [–4,4]?
  A: _____
  _____

- *Example. Find and identify any critical points for* $f(x) = e^{-x} - e^{-2x}$ *on* [0,1].