

Section 3.7 – Indeterminate Forms and L'Hopital's Rule

Recall Limits:

- We were working with limits in Chapters 1 and 2. Recall when we encountered a $0/0$ in a rational expression, we could perhaps “fix” the behavior and analyze the limit by factoring and canceling terms.

- *Example.* Find $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

- Now we have an ‘easier’ way... If we are taking the limit and run across $0/0$, we can take the derivative of the top and bottom and try again.

- *Example above (reworked)*

- This is what is known as applying L'Hopital's Rule.

Indeterminate Forms:

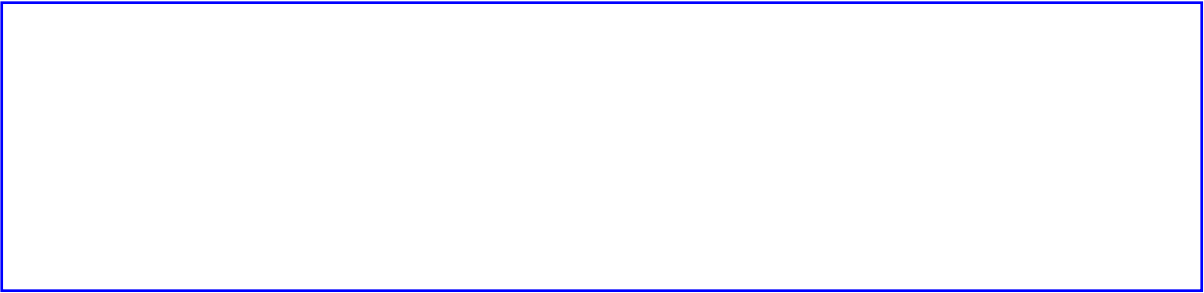
- The following forms are ‘indeterminate’ meaning we are not sure what happens and need to investigate further:

$$\frac{0}{0} \quad \frac{\infty}{\infty} \quad 0 \cdot \infty \quad \infty - \infty \quad 0^0 \quad \infty^0 \quad 1^\infty$$

- L'Hopital's Rule applies when you have either of the indeterminate forms $\left(\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right)$.
- You can use L'Hopital's Rule more than once so long as you still have the indeterminate form above.

- *Example.* Evaluate $\lim_{x \rightarrow -2} \frac{x + 2}{x^2 + 3x + 2}$

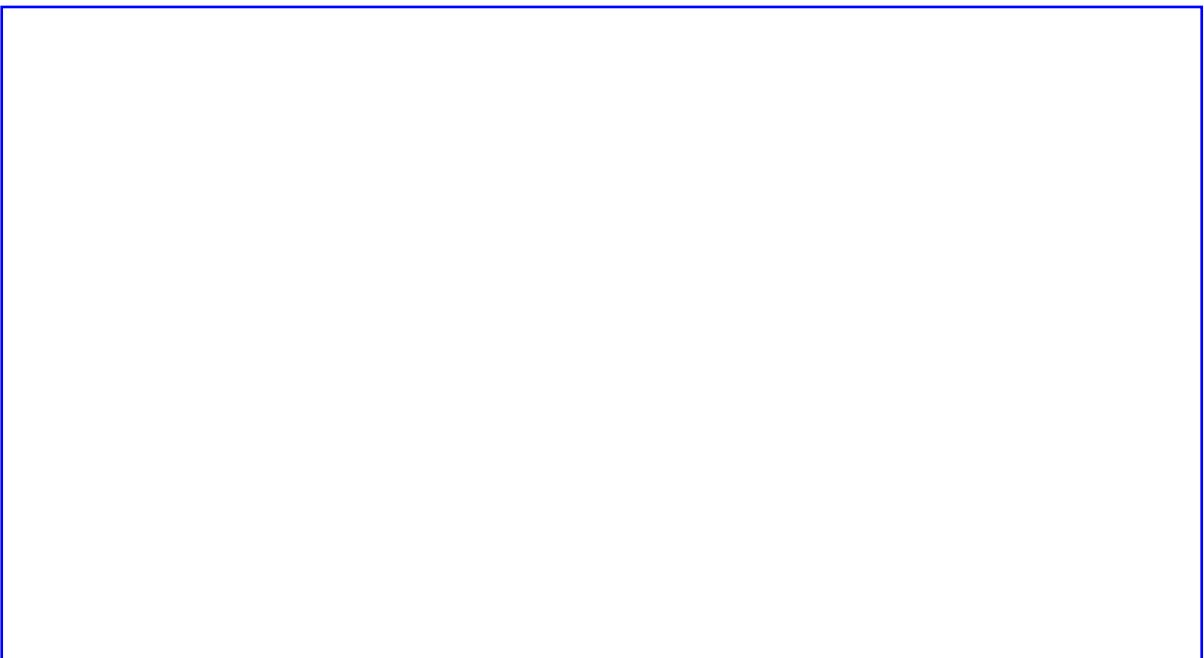
- *Example. Evaluate* $\lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{\csc x}$



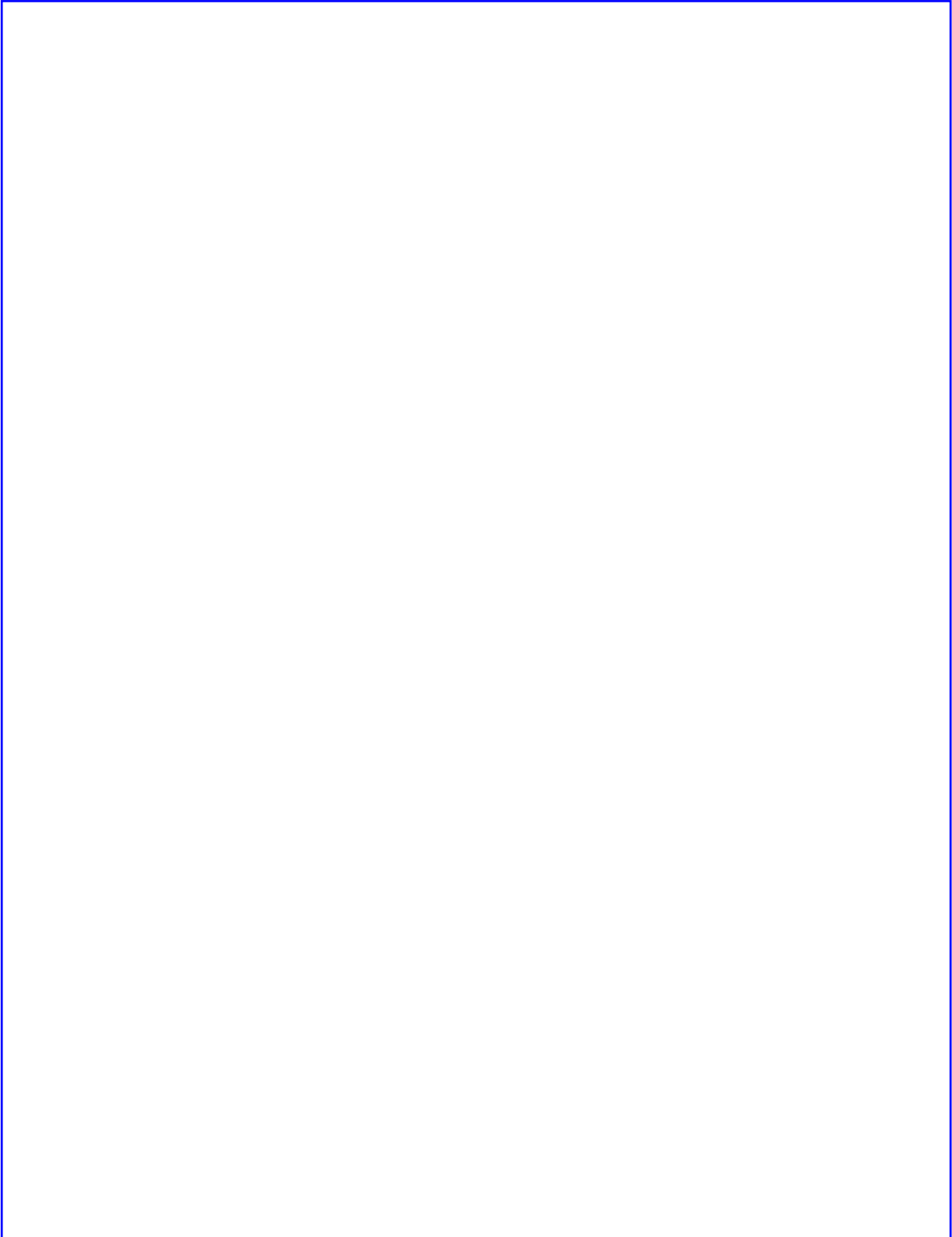
- *Example. Evaluate* $\lim_{x \rightarrow -\infty} x^2 e^x$.



- *Example. Evaluate* $\lim_{x \rightarrow \infty} x \cdot \sin\left(\frac{1}{x}\right)$



- *Example.* Evaluate $\lim_{x \rightarrow 0} (\csc x - \cot x)$



- *Example. Evaluate* $\lim_{x \rightarrow \infty} \left(\frac{2x-3}{2x+5} \right)^{2x+1}$

