Section 3.7 – Indeterminate Forms and L'Hopital's Rule

Recall Limits:

- We were working with limits in Chapters 1 and 2. Recall when we encountered a 0/0 in a rational expression, we could perhaps "fix" the behavior and analyze the limit by factoring and canceling terms.

- *Example. Find* $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

- Now we have an 'easier' way… If we are taking the limit and run across 0/0, we can take the derivative of the top and bottom and try again.

- *Example above (reworked)*

- This is what is known as applying L'Hopital's Rule.

Indeterminate Forms:

- The following forms are 'indeterminate' meaning we are not sure what happens and need to investigate further:

$$\frac{0}{0} \quad \frac{\infty}{\infty} \quad 0 \cdot \infty \quad \infty - \infty \quad 0^0 \quad \infty^0 \quad 1^\infty$$

- L'Hopitals Rule applies when you have either of the indeterminate forms $\left(\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right)$.

- You can use L'Hopitals Rule more than once so long as you still have the indeterminate form above.

- *Example. Evaluate* $\lim_{x \to -2} \frac{x + 2}{x^2 + 3x + 2}$

- *Example. Evaluate* $\lim_{x\to\pi/2} \frac{1-\sin x}{\csc x}$

- *Example. Evaluate* $\lim_{x\to-\infty} x^2 e^x$.

- *Example. Evaluate* $\lim_{x\to\infty} x \cdot \sin\left(\frac{1}{x}\right)$

- *Example. Evaluate* $\lim_{x\to 0}(\csc x - \cot x)$

- *Example. Evaluate* $\lim_{x\to\infty}\left(\frac{2x-3}{2x+5}\right)^{2x+1}$