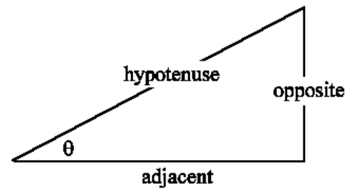
Section 3.5 – Inverse Trig Functions

Recall the Trigonometric Functions:

- The six trig functions are defined in terms of right triangles (see figure).
- The two "most important" being sine and cosine.

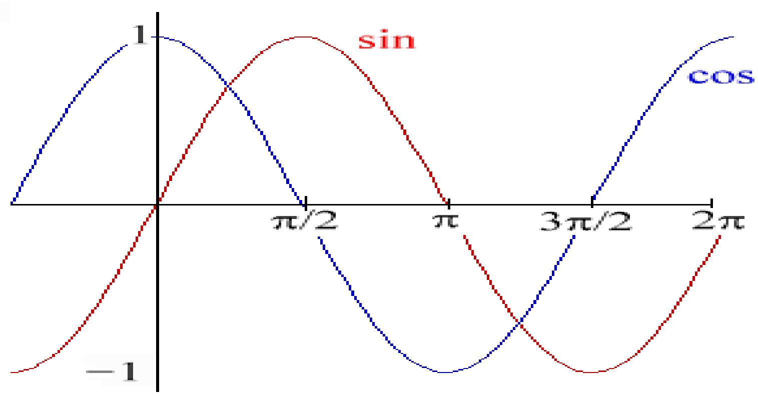
$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$$
$$\cos\theta = \frac{\text{adj.}}{\text{hyp.}}$$



- All other trig functions can be defined in terms of sine and cosine, so remember the definitions above, and the relationships between them and the others…

$$\sec\theta = \frac{1}{\cos\theta} \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\csc\theta = \frac{1}{\sin\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

- Evaluating trig functions boils down to some fundamentals.

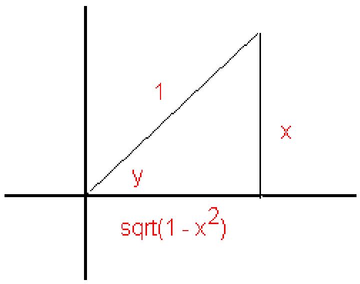


- First, remember the basic shape of sin(x) and cos(x), along with when they are 0, and ±1. From the figure you can see they are periodic, both with a period of 2π.

The Inverse Trig Functions:

- $\sin^{-1} x = y$ *iff* $\sin y = x$ and $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
- $\cos^{-1} x = y$ *iff* $\cos y = x$ and $0 \le y \le \pi$
- $\tan^{-1} x = y$ *iff* $\tan y = x$ and $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
- Note the domain restriction for the inverse trig functions.

- To be able to determine the derivative of sine inverse, we first must recall some trig… If we want to graph $\sin y = x$ we would have



This would mean that $\cos(y) = \sqrt{1-x^2}$

Using implicit differentiation, we find

$$\frac{d}{dx}(\sin y = x)$$
$$y' \cos y = 1$$
$$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

<u>The Inverse Trig Functions:</u>

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- $\cos^{-1} x = y$ *iff* $\cos y = x$ and $0 \le y \le \pi$
- $\tan^{-1} x = y$ *iff* $\tan y = x$ and $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
- Note the domain restriction for the inverse trig functions.
- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

  $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

  $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ $\qquad$ $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

<u>*Some Example Problems:*</u>

- *Example. Find the value of* $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ *and use it to solve* $\cos x = \frac{\sqrt{3}}{2}$ *for x in [0, 2π).*

- *Example. Find the derivative of* $y = \cos^{-1}(e^x + 2x)$.

- *Example. Find the derivative of* $y = \sin^{-1}\left(\frac{x+1}{x-1}\right)$