

Sections 3.1 and 3.4 – Exponential Functions (Growth and Decay)

What Would You Rather Have...

- \$1million, or double your money every day for 31 days starting with 1cent?

Day	Cents	Day	Cents
0	1	16	65536
1	2	17	131072
2	4	18	262144
3	8	19	524288
4	16	20	1048576
5	32	21	2097152
6	64	22	4194304
7	128	23	8388608
8	256	24	16777216
9	512	25	33554432
10	1024	26	67108864
11	2048	27	134217728
12	4096	28	268435456
13	8192	29	536870912
14	16384	30	1073741824
15	32768	31	2147483648

- On day 31, you will have 2,147,483,648 cents, or \$21,474,836.48
- **Q: How can we express this with an equation?**
A: _____.
- **Q: From the table, on which day will you surpass \$1million, or 10^8 cents?**
A A: _____.

Exponential Equations Arise When:

- You add the same percentage to a quantity each fixed time period.
- You multiply a quantity by the same amount each fixed time period.
- For the money example, we were adding 100% each day, or multiplying by 2.

Recall:

- One form of the Exponential Function is $y = k b^t$
 b is the base, and k is the initial quantity (when $t = 0$).
- **Q: What are the conditions on b ?**
A: _____.
- The book uses another form of the exponential equation, $y = a^t$.
Q: What is different in these forms?
A: _____.

- Sometimes we talk about the exponential form of the equation as $y(t) = P_0 e^{kt}$. Be careful of the difference between the forms $P_0 e^{kt}$ and $k b^t$. Sometimes it is standard when one says ‘percentage change’ that they are explicitly giving the value of k in $P_0 e^{kt}$ (as is the case in your textbook) but this typically refers to the change in a population, which is *continuously* growing. It wouldn’t be the case for, say, interest which can grow monthly or weekly.

- **Q: In the equations $P_0 e^{kt}$ and $k b^t$, what is the relationship between P_0 and k , and k and b ?**

A: _____.

- *Rules:*

$$b^x b^y = b^{x+y}$$

$$(b^x)^y = b^{xy}$$

$$b^{-x} = \frac{1}{b^x}$$

The Relationship between Equations:

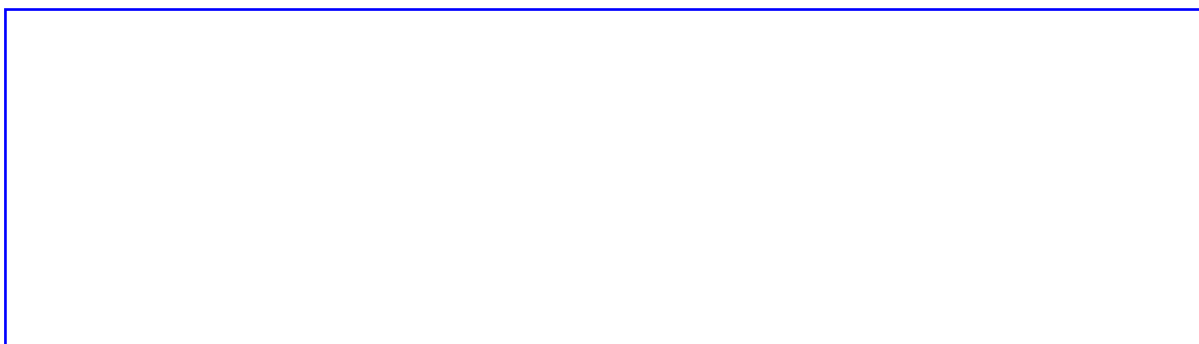
- When you compare the equations $A \left(1 + \frac{r}{n}\right)^{nt}$ and $P_0 e^{kt}$ and $k b^t$, you can see a similarity but be careful which is used.
- Interest problems are of the form $A \left(1 + \frac{r}{n}\right)^{nt}$, where r is the interest rate, A the original amount of money, and n is the number of times you compound per year. It is assumed in these problems that n is compounded a fixed number of times per year, *not* continuously.
- When money (or a population) grows *continuously* the equation $A \left(1 + \frac{r}{n}\right)^{nt}$ changes to $A e^{rt}$. To see the relationship here, note that $\lim_{\frac{n}{r} \rightarrow \infty} \left(1 + \frac{1}{(n/r)}\right)^{n/r} = e$. Your growth rate is then equal to r .
- Provided you are dealing with continual growth, you can use either $P_0 e^{kt}$ or $k b^t$. If the growth rate r is given,

Using $k b^t$, $b = e^r$

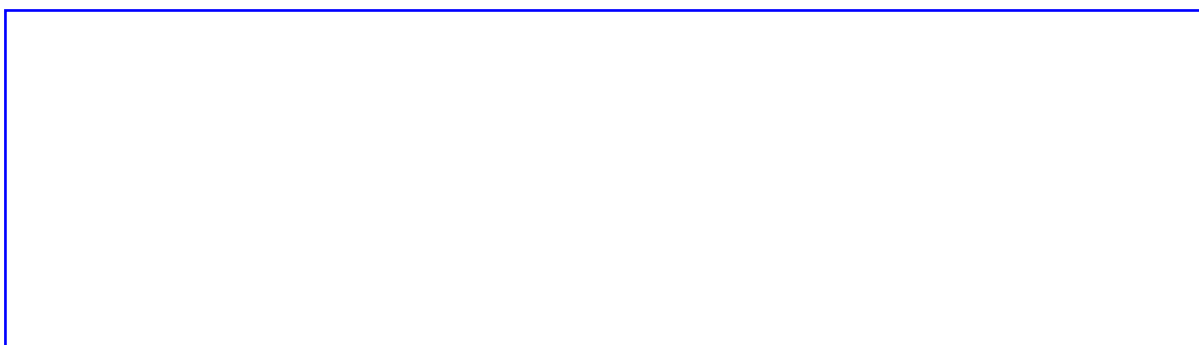
Using $P_0 e^{kt}$, $k = r$.

Graphs of Exponentials:

- $y = k b^t$
 - If the base is greater than 1, you have _____ .
 - If the base is less than 1, you have _____ .
 - The y intercept is k .
 - If $k > 0$, the function will always be positive. As t gets large, the function will tend to 0 (for decay) and infinity (for growth).
 - If $k < 0$, the function will always be negative (it is flipped upside down).
- If you have an equation that is exponential in form, but are adding or subtracting a constant
 - $y = k b^t + c$, this is a shift up ($c > 0$) or down ($c < 0$) by c
 - $y = k b^{(t+c)}$, this is a shift left ($c < 0$) or right ($c > 0$) by c
- *Example. Graph $y = 3^{-x}$*



- *Example. Graph $f(x) = 2 - e^{-x}$*



Testing a Data Set (OPTIONAL):

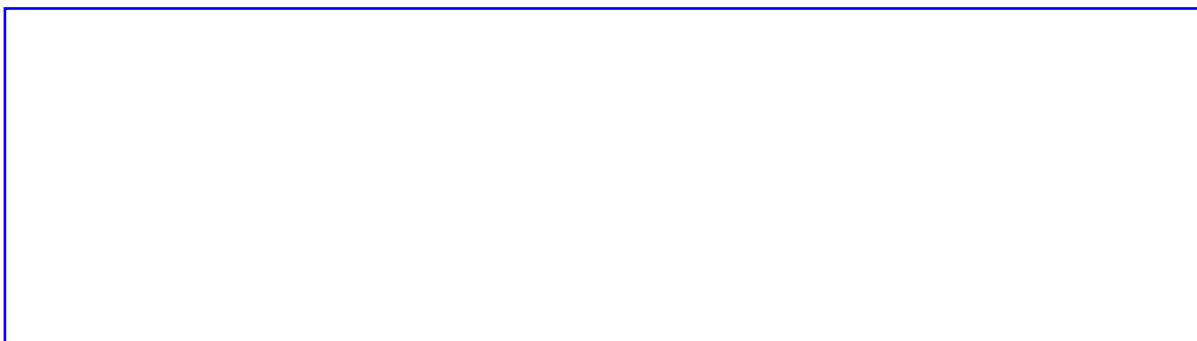
t	y	y2 / y1
0	2	
1	6	3
2	18	3
3	54	3
4	162	3

- Determine if the following set of data is exponential by taking divisions of successive y values
- For each fixed time period (change in t is one) we seem to be multiplying by 3.
- This means that $b = (y2 / y1) / (\text{change in } t) = 3$, and $k = y(0) = 2$.
- $y(t) = 2 \cdot 3^t$

- *Example. If a species of bacteria has a doubling time of 45 minutes, then find b in $k b^t$.*



- *Example. If a species of bacteria has a doubling time of 45 minutes, then find k in $P_0 e^{kt}$.*



- *Example: The world population was 2560 million in 1950 and 3040 million in 1960. Model the population with an exponential equation.*

