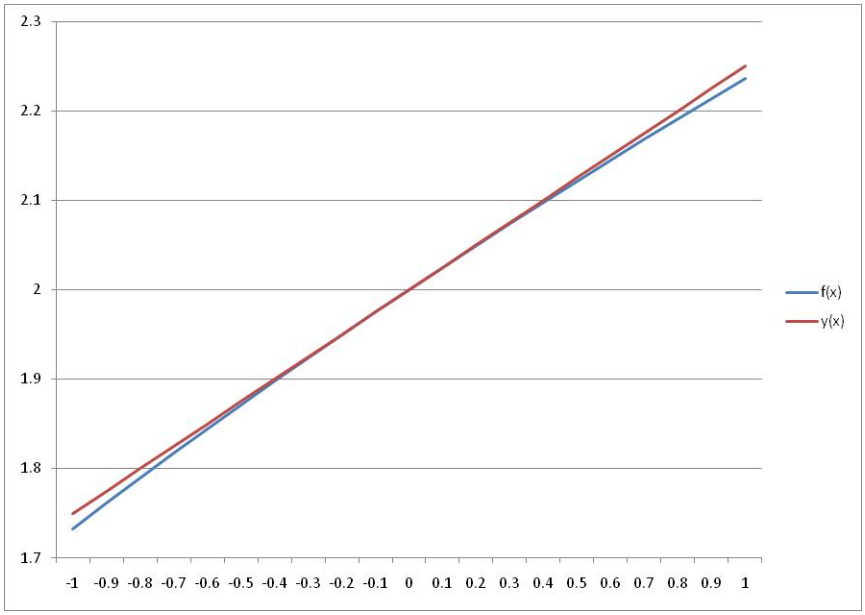
Section 2.8 – Linear Approximations

The Graph of the Tangent Line:

- Let's say we want to estimate the value for $\sqrt{4.1}$. We know this will be near the value for $\sqrt{4} = 2$.
- So let's look at the function $f(x) = \sqrt{x+4}$, along with the equation of the tangent line at $x = 0$.



- Q: What is the equation of the tangent line?
  A:

- If we did not know the values of $f(x)$ near $x = 0$, that is, something like $\sqrt{4.1}$, $\sqrt{4.2}$, $\sqrt{3.9}$, etc., we could use this approximation. Looking at some values we can see how close we are

x	f(x)	y(x)
-1	1.732051	1.75
-0.9	1.760682	1.775
-0.8	1.788854	1.8
-0.7	1.81659	1.825
-0.6	1.843909	1.85
-0.5	1.870829	1.875
-0.4	1.897367	1.9
-0.3	1.923538	1.925
-0.2	1.949359	1.95
-0.1	1.974842	1.975
0	2	2
0.1	2.024846	2.025
0.2	2.04939	2.05
0.3	2.073644	2.075
0.4	2.097618	2.1
0.5	2.12132	2.125
0.6	2.144761	2.15
0.7	2.167948	2.175
0.8	2.19089	2.2
0.9	2.213594	2.225
1	2.236068	2.25

- The tangent line given by $L(x) = f(a) + f'(a)(x-a)$ when it is used this way is called the _____ of f at a.
- Q: What is the percentage error if we use this idea to find $\sqrt{4.2}$?
  A:
- Q: What is the percentage error if we use this idea to find $\sqrt{3}$?
- Notice that the further away we go from the original center point (for the tangent line), the worse our approximation.
- In our example above, the linearization of f at $x = 2$ is given by

$$L(x) = f(a) + f'(a)(x-a) \Rightarrow L(x) = 2 + \frac{1}{4}(x-0) = 2 + \frac{1}{4}x$$