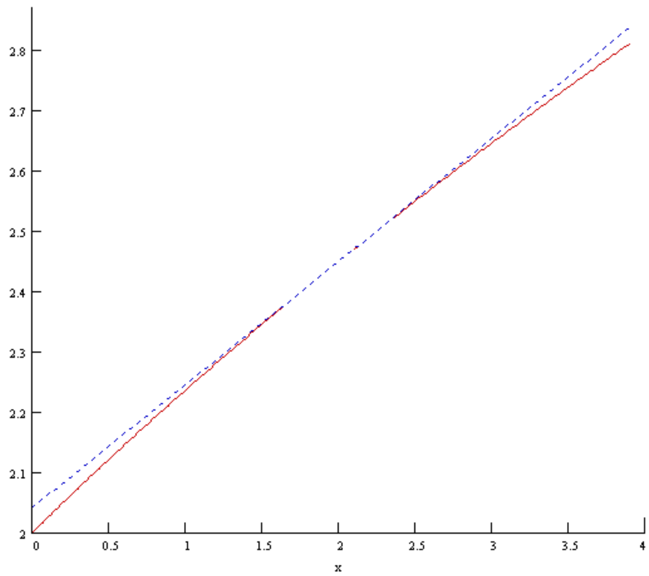
Section 2.8 – Linear Approximations

The Graph of the Tangent Line:

- For the function $f(x)=\sqrt{x+4}$, we find $f'(x)=\frac{1}{2}(x+4)^{-1/2}$.
- Let's look at the graph of the function along with the graph of the tangent line at $x = 2$.
- Note: $f(2)=\sqrt{6},\ f'(2)=\frac{1}{2\sqrt{6}}$



- Q: What is the equation of the tangent line?
  A: Using the information above to find the equation of the line $g(x)$

  $$g(x)-f(2)=m(x-2)$$

  $$g(x)-\sqrt{6}=\frac{1}{2\sqrt{6}}(x-2)$$

  $$g(x)=\frac{1}{2\sqrt{6}}x+\frac{5}{\sqrt{6}}$$

- If we did not know the values of $f(x)$ near $x = 2$, we could use this approximation. Looking at some values we can see how close we are

x	**f(x)**	**g(x)**
1.5	2.34521	2.34743
1.6	2.36643	2.36784
1.7	2.38747	2.38825
1.8	2.40832	2.40866
1.9	2.42899	2.42908
2	2.44949	2.44949
2.1	2.46982	2.4699
2.2	2.48998	2.49031
2.3	2.50998	2.51073
2.4	2.52982	2.53114
2.5	2.54951	2.55155

- The tangent line given by $L(x) = f(a) + f'(a)(x-a)$ when it is used this way is called the **linearization** of f at a.
- Q: What is the percentage error at $x = 1.9$?

  A: For percentage error, we use the formula $\left|\frac{\text{estimate} - \text{original}}{\text{original}} \times 100\%\right|$

  $$\left|\frac{2.42098 - 2.42899}{2.42899} \times 100\%\right| = 0.003298 \times 100\% = 0.3298\%$$

- In our example above, the linearization of f at $x = 2$ is given by

  $$L(x) = f(a) + f'(a)(x-a) = \sqrt{6} + \frac{1}{2\sqrt{6}}(x-2) = \frac{5}{\sqrt{6}} + \frac{1}{2\sqrt{6}}x$$