

Section 2.6 – Implicit Differentiation

More Limitations:

- So far we have seen only functions defined explicitly in terms of one variable. For example,
 $f(x) = x \cdot \cos(x^2)$
- Some functions are not defined explicitly, and the second variable (the f in the above case) is hidden *inside* the relation itself. That is $f^2 + 2x = \cos x$. These are called _____.
- It is still advantageous to find the derivative of such relations, without solving for the variable explicitly.

The Chain Rule in Disguise:

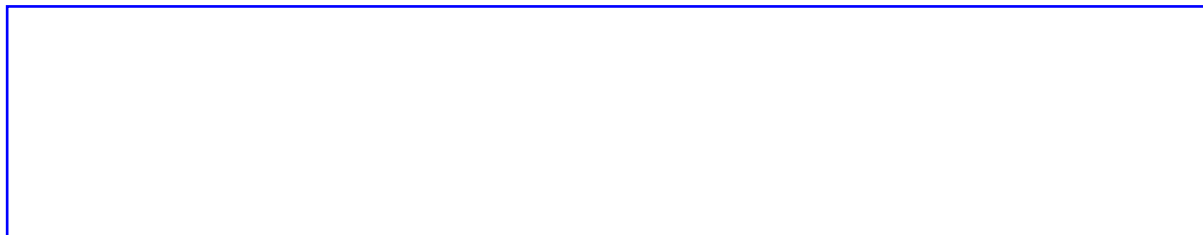
- What you need to remember is that in the example above, f is actually a function, and should be treated as such. Which means you need to use the chain rule when differentiating it.
- For example, let's look at the problem below with several different $y(x)$ functions defined in the [] and see if we can see the pattern:

$$\left([x^2 + 3x]^2 + 5x = 2[x^2 + 3x] \right)' \Rightarrow 2[x^2 + 3x]^1 (x^2 + 3x)' + 5 = 2 \left[(x^2 + 3x)' \right]$$

$$\left([e^{2x}]^2 + 5x = 2[e^{2x}] \right)' \Rightarrow 2[e^{2x}]^1 (e^{2x})' + 5 = 2 \left[(e^{2x})' \right]$$

$$\left([y(x)]^2 + 5x = 2[y(x)] \right)' \Rightarrow 2[y(x)]^1 y'(x) + 5 = 2[y'(x)]$$

- To find the derivative of an implicit relation:
 1. Differentiate the entire line as you would any other function, keeping the = sign in place
 2. Be sure to use the chain rule!
 3. Solve for the derivative (f' , y' , etc.)
- *Example. Find y' if $x + y = 3$*



- *Example. Find y' if $x^2y + \cos(x) = 3$*

- *Example. Find y' if $x^2y^3 + \cos(y) = 3$*

- *Example. Find the derivative of the function $x^2 + 2xy - y^2 + x = 2$ at (1,2)*