Section 2.6 – Implicit Differentiation

More Limitations:

- So far we have seen only functions defined explicitly in terms of one variable. For example, $f(x) = x \cdot \cos(x^2)$
- Some functions are not defined explicitly, and the second variable (the *f* in the above case) is hidden *inside* the relation itself. That is $f^2 + 2x = \cos x$. These are called **implicit relations**.
- It is sill advantageous to find the derivative of such relations, without solving for the variable explicitly.

The Chain Rule in Disguise:

- What you need to remember is that in the example above, *f* is actually a function, and should be treated as such. Which means you need to use the chain rule when differentiating it.
- To find the derivative of an implicit relation:
  1. Differentiate the entire line as you would any other function, keeping the = sign in place
  2. Be sure to use the chain rule!
  3. Solve for the derivative (f', y' , etc.)
- *Example. Find* y' if $x^2 y + \cos(x) = 3$

  Realize that $x^2 y$ is a product, and you need to use the chain rule with *y*

  $x^2(y') + y(2x^1) - \sin(x) = 0$

  $x^2(y') = -2xy + \sin(x)$

  $y' = \dfrac{-2xy + \sin(x)}{x^2}$
- *Example. Find* y' if $x^2 y^3 + \cos(y) = 3$

  $x^2(3y^2 y') + y^3(2x) - \sin(y) \cdot y' = 0$

  $y'(3x^2 y^2 - \sin y) = -2xy^3$

  $y' = \dfrac{-2xy^3}{3x^2 y^2 - \sin y}$
- *Example. Find the derivative of the function* $x^2 + 2xy - y^2 + x = 2$ *at* (1,2)

  $2x + 2(xy' + y) - 2yy' + 1 = 0$

  $y'(2x - 2y) = -2x - 2y - 1$

  $y' = \dfrac{-2x - 2y - 1}{2x - 2y}$

  At (1,2) we have $y' = \dfrac{-2(1) - 2(2) - 1}{2(1) - 2(2)} = \dfrac{7}{2}$