

Section 2.3 – Basic Differentiation Formulas

Some Advice:

- So clearly the ‘old way’ of finding a derivative by evaluating the limit function is just too time consuming.
- Beginning now we will be learning shortcut ways of finding the derivative function that prevent us from having to analyze the limit function.
- **Q: Does the limit function still exist even though we are not finding it?**
A: _____.
- The ideas in this section all build upon each other, and if we develop good habits now we will have a much easier job when the functions become more complicated.
- In that regard, it is important that you understand the formulas presented here in theory... **DO NOT MEMORIZE** the formulas, rather understand their meaning in **WORDS**. These words are highlighted in **yellow**.

Derivative of a Constant Function:

- What is a constant function? $y(x) = c$ for some constant c .
- $\lim_{h \rightarrow 0} \frac{y(x+h) - y(x)}{h} = \lim_{h \rightarrow 0} \frac{c - c}{h} = \lim_{h \rightarrow 0} \frac{0}{h} = \lim_{h \rightarrow 0} 0 = 0$.
- So the derivative of any constant function is 0.
- **Q: What does the graph of a constant function look like?**
A: _____.
- **Q: What would its tangent line look like, and does it indeed have a slope of 0?**
A: _____.
- Instead of memorizing formulas, memorize the concept, in words:
The derivative of a constant is zero

Derivative of Power Function:

- A power function is of the form $y(x) = x^n$.
- $n = 1: \frac{d}{dx}(x^1) = \lim_{h \rightarrow 0} \frac{y(x+h) - y(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^1 - x^1}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = 1$
- $n = 2: \frac{d}{dx}(x^2) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = 2x$
- $n = 3: \frac{d}{dx}(x^3) = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{x^3 + 3xh^2 + 3x^2h + h^3 - x^3}{h} = 3x^2$

- So the pattern is in fact... $\frac{d}{dx}(x^n) = nx^{n-1}$.
- Instead of memorizing formulas, memorize the concept, in words:
The derivative of a function to a power is the power times the function to the one less power
- In general, n can be any real number value (even though our examples were whole numbers).

Some First Rules:

- The derivative of a sum is the sum of the derivatives

$$\frac{d}{dx}[f + g] = \lim_{h \rightarrow 0}[f(x) + g(x)] = \lim_{h \rightarrow 0} f(x) + \lim_{h \rightarrow 0} g(x) = \frac{df}{dx} + \frac{dg}{dx}$$

- The derivative of a constant times a function is the constant times the derivative of that function

$$\frac{d}{dx}[c \cdot f(x)] = \lim_{h \rightarrow 0}[c \cdot f(x)] = c \cdot \lim_{h \rightarrow 0} f(x) = c \cdot \frac{df}{dx}$$

- Combining the first rule with the second (where $c = -1$) we have a 'new' rule

The derivative of a difference is the difference of the derivatives

$$\frac{d}{dx}[f - g] = \lim_{h \rightarrow 0}[f(x) - g(x)] = \lim_{h \rightarrow 0} f(x) - \lim_{h \rightarrow 0} g(x) = \frac{df}{dx} - \frac{dg}{dx}$$

Derivative of the Exponential Function:

- The derivative of the Natural Exponential Function is itself

$$\frac{d}{dx}(e^x) = e^x$$

- Why would this be true? First we need to understand what e is actually...

e is the number so that $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$.

- Using the definition of derivative, we find $\lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \rightarrow 0} \left[e^x \frac{e^h - 1}{h} \right] = e^x \lim_{h \rightarrow 0} \left[\frac{e^h - 1}{h} \right] = e^x(1) = e^x$.

Some Example Problems of Finding the Derivative:

- Again, be sure to say these problems in words. Get used to using the definitions instead of memorizing formulas.

- *Example.* Differentiate $f(t) = \frac{1}{2}t^4 - 3t^2$

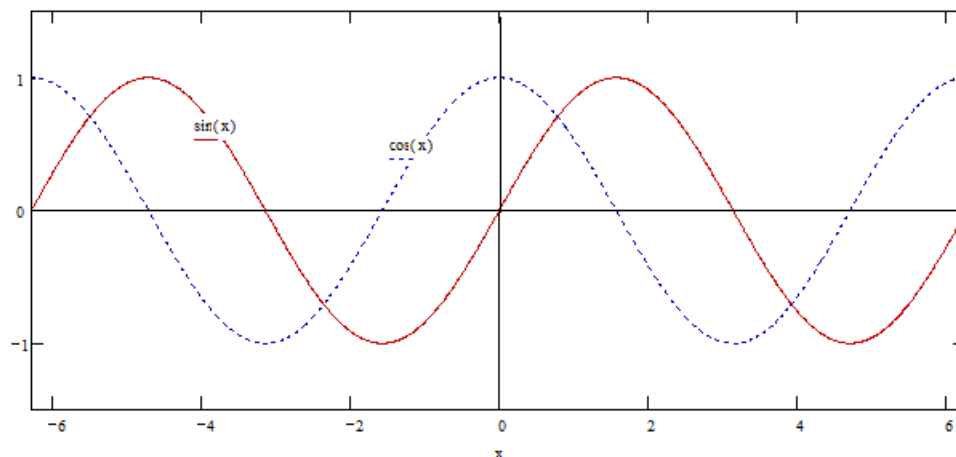
- *Example. Differentiate $g(x) = \sqrt{2}x + \sqrt{x}$*

- *Example. Differentiate $h(x) = \frac{1}{x}$*

- *Example. For what values of x does the graph of $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x$ have a horizontal tangent?*

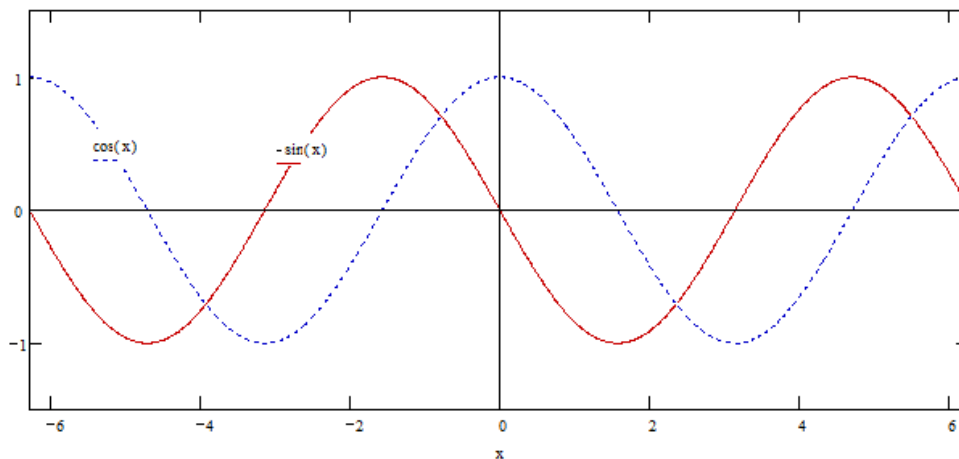
Sine and Cosine Functions:

- First, the angle measured could be in radians or degrees. From now on we will use radians.
- The graph of sine and cosine are



- Q: Where are the roots of $\sin(x)$?
A: _____.
- Q: Where are the horizontal tangents of $\sin(x)$?
A: _____.
- Q: Where are the roots of $\cos(x)$?
A: _____.
- Q: Where are the horizontal tangents of $\cos(x)$?
A: _____.
- Notice that $\sin(x)$ has a horizontal tangent everywhere $\cos(x)$ has a root. Also notice that cosine is positive when sine is increasing, and cosine is negative when sine is decreasing. We've just seen that the derivative of sine is actually cosine!
- $\frac{d}{dx} \sin(x) = \cos(x)$.
The derivative of sine is cosine

- Now let's compare the cosine and $-\sin$ functions



- Notice that $\cos(x)$ has a horizontal tangent everywhere $-\sin(x)$ has a root. Also notice that when cosine is increasing, $-\sin(x)$ is positive, and when cosine is decreasing, $-\sin(x)$ is negative. The derivative of cosine is $-\sin$!
- $\frac{d}{dx} \cos(x) = -\sin(x)$.
The derivative of cosine is negative sine