# Section 2.3 – Basic Differentiation Formulas

*Some Advice:*

- So clearly the 'old way' of finding a derivative by evaluating the limit function is just too time consuming.
- Beginning now we will be learning shortcut ways of finding the derivative function that prevent us from having to analyze the limit function.
- Q: Does the limit function still exist even though we are not finding it?
  A: Yes. And you are still responsible for knowing it, and being able to use it to find the derivative.
- The ideas in this section all build upon each other, and if we develop good habits now we will have a much easier job when the functions become more complicated.
- In that regard, it is important that you understand the formulas presented here in theory… DO NOT MEMORIZE the formulas, rather understand their meaning in WORDS. These words are highlighted in yellow.

*Derivative of a Constant Function:*

- What is a constant function? $y(x) = c$ for some constant $c$.
- $\lim\limits_{h\to 0}\dfrac{y(x+h)-y(x)}{h} = \lim\limits_{h\to 0}\dfrac{c-c}{h} = \lim\limits_{h\to 0}\dfrac{0}{h} = \lim\limits_{h\to 0} 0 = 0$.
- So the derivative of any constant function is 0.
- Q: What does the graph of a constant function look like?
  A: It is a horizontal line.
- Q: What would its tangent line look like, and does it indeed have a slope of 0?
  A: The tangent of this function would be the slope of the line, which is 0.
- Instead of memorizing formulas, memorize the concept, in words:
  The derivative of a constant is zero

*Derivative of Power Function:*

- A power function is of the form $y(x) = x^n$.
- $n = 1$: $\dfrac{d}{dx}(x^1) = \lim\limits_{h\to 0}\dfrac{y(x+h)-y(x)}{h} = \lim\limits_{h\to 0}\dfrac{(x+h)^1 - x^1}{h} = \lim\limits_{h\to 0}\dfrac{h}{h} = 1$
- $n = 2$: $\dfrac{d}{dx}(x^2) = \lim\limits_{h\to 0}\dfrac{(x+h)^2 - x^2}{h} = \lim\limits_{h\to 0}\dfrac{x^2 + 2xh + h^2 - x^2}{h} = 2x$
- $n = 3$: $\dfrac{d}{dx}(x^3) = \lim\limits_{h\to 0}\dfrac{(x+h)^3 - x^3}{h} = \lim\limits_{h\to 0}\dfrac{x^3 + 3xh^2 + 3x^2h + h^3 - x^3}{h} = 3x^2$

- So the pattern is in fact… $\frac{d}{dx}(x^n) = nx^{n-1}$.

- Instead of memorizing formulas, memorize the concept, in words:
  The derivative of a function to a power is the power times the function to the one less power

- In general, $n$ can be any real number value (even though our examples were whole numbers).

*Some First Rules:*

- The derivative of a sum is the sum of the derivatives
  $$\frac{d}{dx}[f+g] = \lim_{h\to 0}[f(x)+g(x)] = \lim_{h\to 0} f(x) + \lim_{h\to 0} g(x) = \frac{df}{dx} + \frac{dg}{dx}$$

- The derivative of a constant times a function is the constant times the derivative of that function
  $$\frac{d}{dx}[c\cdot f(x)] = \lim_{h\to 0}[c\cdot f(x)] = c\cdot \lim_{h\to 0} f(x) = c\cdot \frac{df}{dx}$$

- Combining the first rule with the second (where $c = -1$) we have a 'new' rule
  The derivative of a difference is the difference of the derivatives
  $$\frac{d}{dx}[f-g] = \lim_{h\to 0}[f(x)-g(x)] = \lim_{h\to 0} f(x) - \lim_{h\to 0} g(x) = \frac{df}{dx} - \frac{dg}{dx}$$

*Derivative of the Exponential Function:*

- The derivative of the Natural Exponential Function is itself
  $$\frac{d}{dx}(e^x) = e^x$$

- Why would this be true? First we need an understand what $e$ is actually…
  $e$ is the number so that $\lim_{h\to 0}\frac{e^h - 1}{h} = 1$.

- Using the definition of derivative, we find $\lim_{h\to 0}\frac{e^{x+h} - e^x}{h} = \lim_{h\to 0}\left[e^x\,\frac{e^h-1}{h}\right] = e^x \lim_{h\to 0}\left[\frac{e^h-1}{h}\right] = e^x(1) = e^x$.

*Some Example Problems of Finding the Derivative:*

- Again, be sure to say these problems in words. Get used to using the definitions instead of memorizing formulas.

- *Example. Differentiate* $f(t) = \frac{1}{2}t^4 - 3t^2$
  $$f'(t) = \frac{1}{2}(4)t^3 - 3(2)t^1$$
  $$= 2t^3 - 6t$$

- *Example. Differentiate* $g(x)=\sqrt{2}\,x+\sqrt{x}$

  First notice that $g(x)=\sqrt{2}\,x^1+x^{1/2}$

  $$g'(x)=\sqrt{2}\,(1)x^0+\frac{1}{2}x^{-1/2}$$
  $$=\sqrt{2}+\frac{1}{2\sqrt{x}}$$

- *Example. Differentiate* $h(x)=\dfrac{1}{x}$

  $$h'(x)=\frac{d}{dx}(x^{-1})$$
  $$=-1x^{-2}$$
  $$=\frac{-1}{x^2}$$

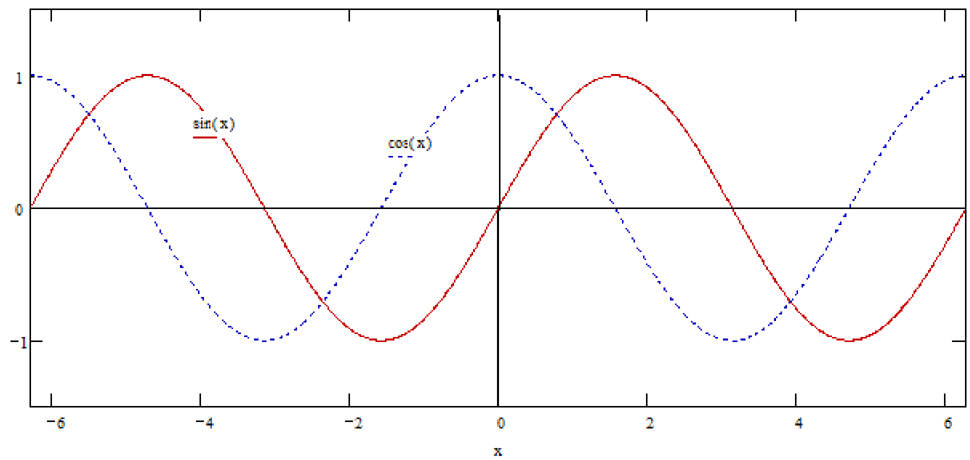
- *Example. For what values of x does the graph of* $f(x)=\dfrac{1}{3}x^3+\dfrac{1}{2}x^2-2x$ *have a horizontal tangent?*

  $$f'(x)=\frac{1}{3}(3)x^2+\frac{1}{2}(2)x^1-2(1)x^0$$
  $$=x^2+x-2$$
  $$=(x-2)(x+1)$$

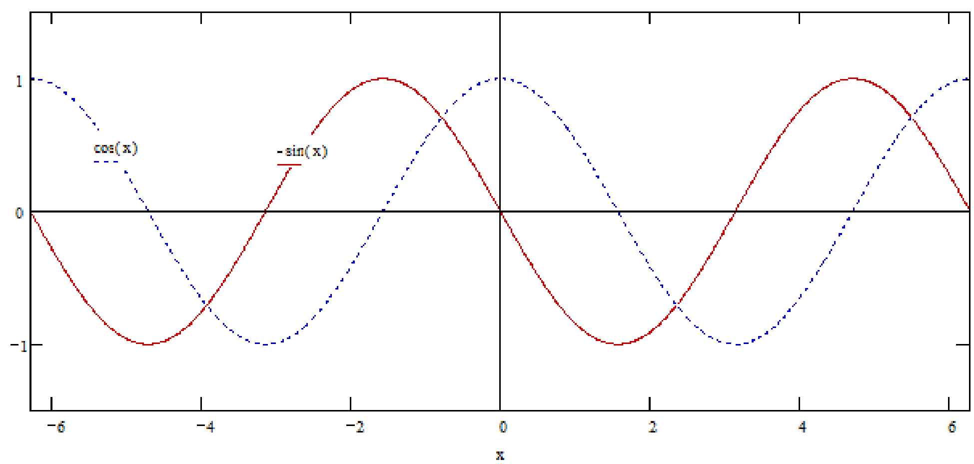
  $f'(x)=0$ when $x$ = 2 and –1

*<u>Sine and Cosine Functions:</u>*

- First, the angle measured could be in radians or degrees. From now on we will use radians.
- The graph of sine and cosine are



- Q: Where are the roots of sin($x$)?
  A: sine is zero at all multiples of pi $(0, \pm\pi, \pm 2\pi ...)$
- Q: Where are the horizontal tangents of sin($x$)?
  A: All odd multiples of pi over 2 $\left(\frac{\pm\pi}{2}, \frac{\pm 3\pi}{2}, \frac{\pm 5\pi}{2} ...\right)$
- Q: Where are the roots of cos($x$)?
  A: cosine is zero at all multiples of pi over 2 $\left(\frac{\pm\pi}{2}, \frac{\pm 3\pi}{2}, \frac{\pm 5\pi}{2} ...\right)$
- Q: Where are the horizontal tangents of cos($x$)?
  A: All odd multiples of pi $(0, \pm\pi, \pm 2\pi ...)$
- Notice that sin($x$) has a horizontal tangent everywhere cos($x$) has a root. Also notice that cosine is positive when sine is increasing, and cosine is negative when sine is decreasing. We've just seen that the derivative of sine is actually cosine!
- $\frac{d}{dx}\sin(x) = \cos(x)$.
  The derivative of sine is cosine
- Now let's compare the cosine and –sine functions




- Notice that cos($x$) has a horizontal tangent everywhere –sin($x$) has a root. Also notice that when cosine is increasing, –sin($x$) is positive, and when cosine is decreasing, –sin($x$) is negative. The derivative of cosine is –sine!
- $\frac{d}{dx}\cos(x) = -\sin(x)$.
  The derivative of cosine is negative sine