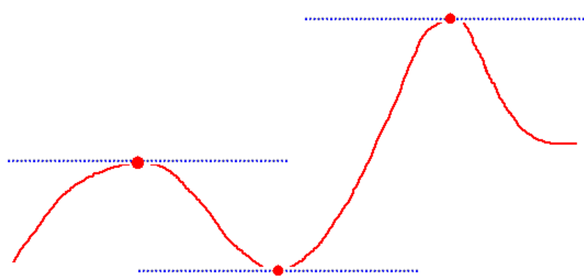
Section 2.2 – The Derivative as a Function

The Derivative as a Function:

- Recall that we defined $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.
- Of course, this limit must exist to work properly.
- If the limit exists at $x = a$, then the function is said to be **differentiable** at $x = a$.
- If the limit exists on some interval (specifically, for all values within that interval) then the function is said to be **differentiable on that interval**.
- If the limit exists for all values in its domain, then the function is said to be **differentiable**. This is the same concept as continuous, where if no point or interval is given, you assume it means everywhere.
- So long as the limit definition above exists, it can be interpreted as a function itself, and it is useful to help us understand the behavior of $f(x)$.
- There are several other symbols used for the derivative of a function, including $f'(x) = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f(x)$.
- Of course, if the function is not f, say it is $y(x)$, then the above definitions would change accordingly.

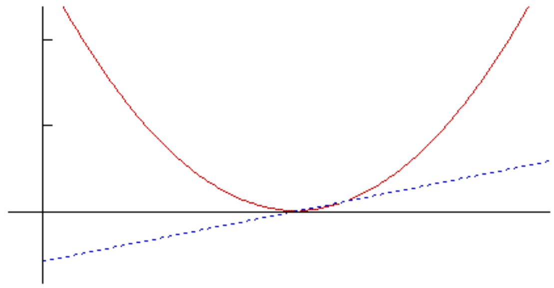
Critical Points:



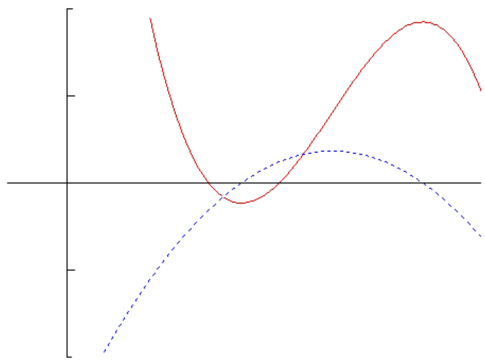
- Q What do you notice about the tangent lines given in the above picture?
  A: The tangent lines at those points are horizontal.

  Q: What is so special about these points?
  A: These points are at the top (or bottom) of a 'hill'.
- We will cover this in more detail later, but the above points are examples of **critical points**, or points where the derivative is zero or undefined. Above, these critical points have a first derivative of zero.
- You can 'match' a graph of a function with its derivative by looking at the critical points, whenever you have a critical point on your original function, the derivative function will be zero (or undefined) there.

- Q: Can you identify which is the original function and which is the derivative?
  A: At the critical point (on the red line which is the function), you see it is a minimum, which corresponds to a zero for the derivative (the blue line).

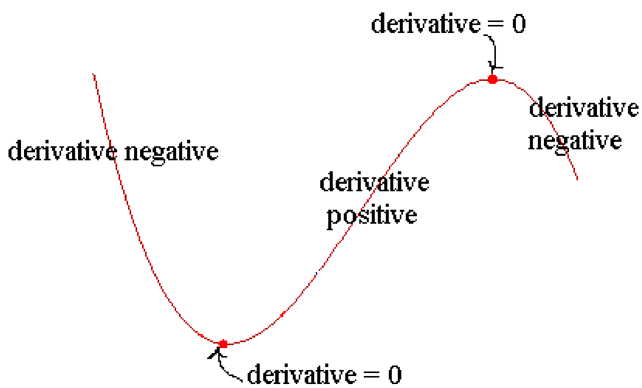


- Q: Can you identify which is the original function and which is the derivative?
  A: Notice that the original function (the red line) has a critical point (it is a minimum) where the blue line (the derivative) has a root, and has a maximum where the blue line has another root.



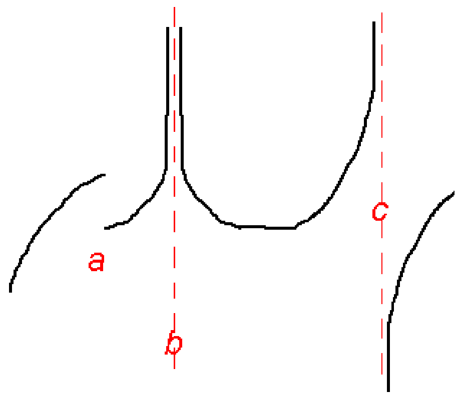
Sign of the derivative:

- When a function is increasing, the slope of the tangent lines will be positive, hence the derivative is **positive** when a function is **increasing**.
- In a similar manner, the derivative is **negative** when a function is **decreasing**.
- When a function changes from increasing to decreasing, you have a **critical point**, where the derivative **goes to zero (or is undefined)**.



<u>What Can Go Wrong:</u>

- Recall the requirements for continuity.
- If a function is not continuous, then it cannot be differentiable. So the following is not differentiable at the values *a*, *b* and *c*.



- Differentiability is actually more strict than continuity.
- Any sharp corners will not be differentiable

- Also, there may be places where the function has a vertical tangent.