Section 1.6 – Limits at Infinity

Recall:

- You have already studied ____________________, that is $f(x)=\dfrac{p(x)}{q(x)}$.
- You should already be able to identify the domain of this function, in other words, the values of *x* that are/are not allowed for inputs of *f*.
- Q: How do we find any undefined points in the domain?
  A:____.
- These points, if they are not ____________________, are called ____________________, and the limit of the function there would tend to plus or minus infinity.
- These types of infinite discontinuities are also called ____________________.

Example. For $f(x)=\dfrac{1}{x-2}$ *What is the limit as x tends to 2 from the right? From the left?*

Horizontal Asymptotes:

- Think of an asymptote in general being an invisible line that a function 'tends' to if you were to keep drawing. You never quite get there.
- For example, in the above function, the graph (as *x* tends to 2) gets closer and closer to the vertical line at *x* = 2 but never quite reaches it. And it will not cross over, either.
- The above example function has another asymptote that is horizontal… If you were to let *x* get larger and larger ($x \to \pm\infty$) the function would get closer and closer to an 'invisible' horizontal line.
- Q: What value does the function tend to as $x \to \pm\infty$? What is the equation of this line?
  A:____.

Finding Horizontal Asymptotes:

- As we 'extend' the function to the left and right as far as we want, we are essentially taking the limit as *x* tends to plus and minus infinity.
- To find the horizontal asymptote, if it exists, take the limit as *x* tends to plus and minus infinity.
- **HOW TO** find horizontal asymptotes for rational functions $f(x) = \dfrac{p(x)}{q(x)}$:

1. Find the leading term of the numerator, $p(x)$ and denominator, $q(x)$.
2. Write as a quotient and simplify.
3. If the reduction is
   – a constant, then this is the value of your horizontal asymptote.
   – $\text{constant} \cdot \left(\dfrac{1}{x^{\text{positive value}}} \right)$, then the horizontal asymptote is zero
   – $\text{constant} \cdot (x^{\text{positive value}})$, then there is no horizontal asymptote
   It will tend to plus or minus infinity (plug in to see which).

- *Example. Find* $\lim_{x \to \infty} \dfrac{3x^2 + x + 5}{x^2 - 4}$

- *Example. Find* $\lim_{x \to \infty} \dfrac{3x^2 + x^3 + 5}{2x^2 + x^4 - 4}$

- *Example. Find* $\lim_{x \to \infty} \dfrac{3x^2 + x^3 + 5}{2x^2 - 4}$

- *Example. Find* $\lim_{x \to -\infty} \dfrac{x^2(x-1)(x+2)}{3-x}$