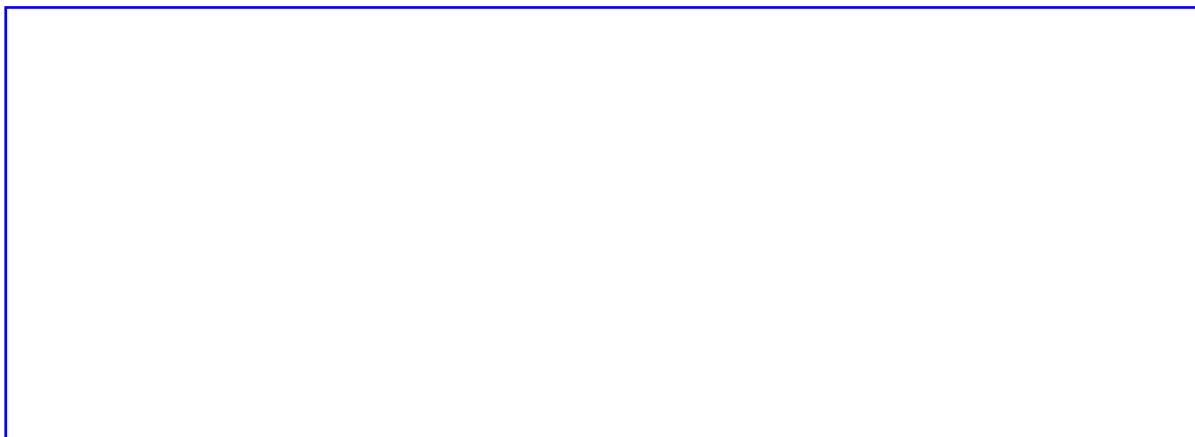


Section 1.6 – Limits at Infinity

Recall:

- You have already studied _____, that is $f(x) = \frac{p(x)}{q(x)}$.
- You should already be able to identify the domain of this function, in other words, the values of x that are/are not allowed for inputs of f .
- **Q: How do we find any undefined points in the domain?**
A: _____.
- These points, if they are not _____, are called _____, and the limit of the function there would tend to plus or minus infinity.
- These types of infinite discontinuities are also called _____.

Example. For $f(x) = \frac{1}{x-2}$ What is the limit as x tends to 2 from the right? From the left?



Horizontal Asymptotes:

- Think of an asymptote in general being an invisible line that a function ‘tends’ to if you were to keep drawing. You never quite get there.
- For example, in the above function, the graph (as x tends to 2) gets closer and closer to the vertical line at $x = 2$ but never quite reaches it. And it will not cross over, either.
- The above example function has another asymptote that is horizontal... If you were to let x get larger and larger ($x \rightarrow \pm\infty$) the function would get closer and closer to an ‘invisible’ horizontal line.
- **Q: What value does the function tend to as $x \rightarrow \pm\infty$? What is the equation of this line?**
A: _____.

Finding Horizontal Asymptotes:

- As we 'extend' the function to the left and right as far as we want, we are essentially taking the limit as x tends to plus and minus infinity.
- To find the horizontal asymptote, if it exists, take the limit as x tends to plus and minus infinity.
- **HOW TO** find horizontal asymptotes for rational functions $f(x) = \frac{p(x)}{q(x)}$:
 1. Find the leading term of the numerator, $p(x)$ and denominator, $q(x)$.
 2. Write as a quotient and simplify.
 3. If the reduction is
 - a constant, then this is the value of your horizontal asymptote.
 - constant $\cdot \left(\frac{1}{x^{\text{positive value}}}\right)$, then the horizontal asymptote is zero
 - constant $\cdot (x^{\text{positive value}})$, then there is no horizontal asymptote
It will tend to plus or minus infinity (plug in to see which).
- *Example. Find $\lim_{x \rightarrow \infty} \frac{3x^2 + x + 5}{x^2 - 4}$*

- *Example. Find $\lim_{x \rightarrow \infty} \frac{3x^2 + x^3 + 5}{2x^2 + x^4 - 4}$*

- *Example.* Find $\lim_{x \rightarrow \infty} \frac{3x^2 + x^3 + 5}{2x^2 - 4}$



- *Example.* Find $\lim_{x \rightarrow -\infty} \frac{x^2(x-1)(x+2)}{3-x}$

