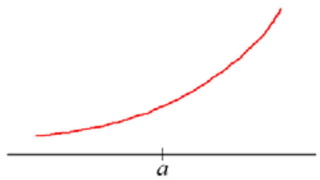
Section 1.5 – Continuity

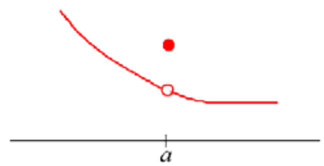
Recall Limits and Function Values:

- We have already studied all the concepts necessary to understand continuity.
- To recap, let's look at the following graphs and answer three questions…
- Q: Does $f(a)$ exist? Does $\lim_{x \to a} f(x)$ exist? Does $\lim_{x \to a} f(x) = f(a)$?

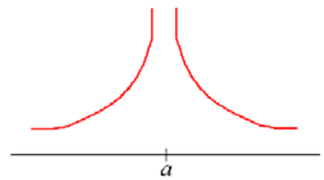
1)



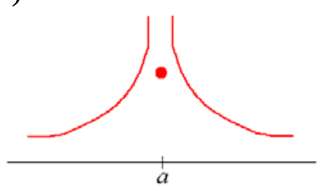
2)



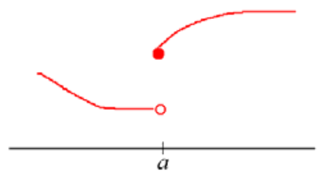
3)



4)



5)



A: 1) ____.

2) ____.

3) ____.

4) ____.

5) ____.

- For a function to be continuous, it has to pass all three tests above, if it fails even one of these tests, it is NOT continuous.
- Q: Which of the above functions are continuous at a?
A: ____.

Formal Definition of Continuity:

- A function f is ____________________ at $x = a$ if and only if
1. $f(a)$ is defined
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$
- If a function f is not continuous at $x = a$, it is said to be ____________________ at $x = a$.

Types of Discontinuities:

- If the limit of the function exists (and is finite) at $x = a$, then continuity fails because either $f(a)$ is not defined, or $\lim_{x \to a} f(x) \neq f(a)$. The discontinuity is classified as ____________________. This is because by simply redefining $f(a)$ we can make f continuous at a.
- Q: Which of the above pictures has a removable discontinuity? How would you 'fix' this?
  A: ____.
- If the limit of the function is infinite, then it is called an ______________________.
- Q: Which of the above pictures has an infinite discontinuity?
  A: ____.
- If the limit from the right and left of a are not the same, then the function is said to have a _______ ________________.
- Q: Which of the above pictures has a jump discontinuity?
  A: ____.

Continuous on an Interval:

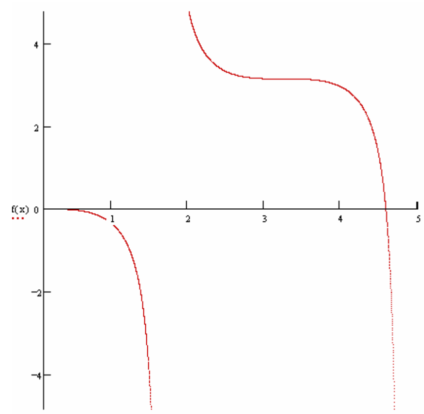
- A function can be __________________________________ at a number (basically you only look at a right or left hand limit).
- A function can be __________________________ if it is continuous for all the points in that interval.
- A function is said to be ____________________ if it is continuous on its entire domain.
- While the same word (i.e. continuous) is used to describe all of these situations, it is important to keep in mind what you are referring to. Here they are from least strict to most strict…
  – continuous at a from the left (or right)
  – continuous at a
  – continuous on an interval
  – continuous
- The following functions are continuous on their domains: polynomials, rational functions, root functions, trig functions, inverse trig functions, exponential functions, log functions.
- But wait, I thought log functions were not defined for negative values?
  Q: How could, say $\log(x)$ satisfy the above statement if it isn't even defined for $x < 0$?
  A: ____.
- Some books (and mathematicians) differ in how they interpret this… some will say a function is not continuous unless it is continuous on the entire real line. Others (like our book) say that as long as the function is continuous on its domain, it is continuous.

Intermediate Value Theorem:

- If f is continuous on the closed interval $[a,b]$ and N is strictly between $f(a)$ and $f(b)$, then there is a number c between a and b such that $f(c) = N$.

- *Example.*
  Let's say you want to find the root of the function $f(x) = x - \tan(x)$. This isn't so easy because it is not an algebraic equation and can't be solved explicitly. We have to 'estimate' the roots.

There are actually an infinite number of them, starting with the trivial root at $x = 0$



So it looks like there is a root somewhere near $x = 4.5$.
We can prove this is true because of the IVT.

f is continuous on [4, 4.6]
$f(4) = 2.8$ and $f(4.6) = -4.26$
$N = 0$ is between 2.8 and –4.26
So by the IVT there is a c between 4 and 4.6 where $f(c) = 0$

We can continue to choose left and right values to move closer and closer to the actual root.