Section 1.4 – Calculating Limits

- In this section we will be using methods to find limits, not prove them as in the last section.
- Sometimes it is helpful to 'break up' limits and work with pieces rather than the whole.
- *Example. Find* $\lim_{x\to 2}\left(2+\frac{1}{(x-2)^2}\right)$

- Most of these rules are intuitive, in that you would think of using them anyway because you are familiar with the rules of addition, subtraction, multiplication and division.

$$\lim_{x\to a} c = c$$

$$\lim_{x\to a} x^n = a^n \qquad \text{if } a=0\ n \text{ must be positive}$$

$$\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a} \qquad a>0 \text{ for } n \text{ even}$$

$$\lim_{x\to a}\left[f(x)\pm g(x)\right] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a} c\cdot f(x) = c\cdot\lim_{x\to a} f(x)$$

$$\lim_{x\to a}\left[f(x)\cdot g(x)\right] = \lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \qquad \lim_{x\to a} g(x)\neq 0$$

$$\lim_{x\to a}\left[f(x)\right]^n = \left[\lim_{x\to a} f(x)\right]^n$$

$$\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} \qquad \lim_{x\to a} f(x)>0 \text{ for } n \text{ even}$$

- A very useful theorem is called the **squeeze theorem**, and it says that if $f(x)\le g(x)\le h(x)$ when x is close to a and $\lim_{x\to a} f(x)=\lim_{x\to a} h(x)=L$, then $\lim_{x\to a} g(x)=L$.

  Essentially what is happening is that *f* and *h* are tending to *L*, and since *g* is squeezed in between them, it must also tend to *L*.

Example Problems:

- *Example. Evaluate* $\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}$

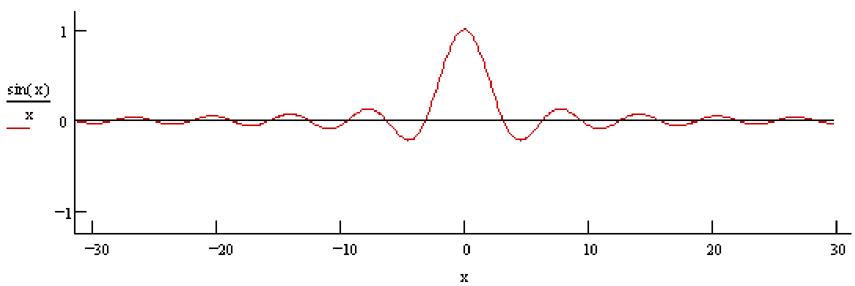
- *Example. Evaluate* $\lim_{h\to 0}\frac{(3+h)^{-1}-3^{-1}}{h}$

- *Example. Evaluate* $\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)$

- *Example. Evaluate* $\lim_{x\to 2}\dfrac{x^2-4}{|x-2|}$

Limits of Trig Functions:

- A famous limit is given by $\lim_{x\to 0}\dfrac{\sin x}{x}=1$.
- This is pretty clear from the graph of the function



- But it can also be found with the squeeze theorem, because near $x = 0$, $\cos x<\dfrac{\sin x}{x}<1$. And both $\lim_{x\to 0}\cos x=1$ and $\lim_{x\to 0}1=1$.

- *Example. Evaluate* $\lim_{x \to 0} \frac{\sin(2x)}{x}$

- *Example. Evaluate* $\lim_{x \to 0} \frac{\cos x - 1}{\sin x}$