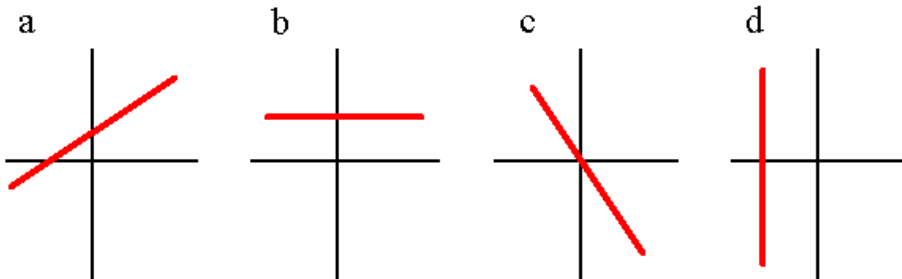


## Section 1.2 – A Catalog of Essential Functions

### Linear Models:

- All linear equations have the form \_\_\_\_\_ .
- The letter  $m$  is the \_\_\_\_\_ of the line,  $\frac{\text{rise}}{\text{run}}$  or  $\frac{\text{change in horizontal}}{\text{change in vertical}}$ . It can be positive, negative or zero. It can also be very large or very small.
- **Q: What would the line look like in each one of these cases?**  
small/positive   large/positive   small/negative   large/negative   zero  
**A:**
- The letters  $x$  and  $y$  are \_\_\_\_\_, meaning they vary or change along the line. At least one of them must be present in the equation. Together they represent the \_\_\_\_\_ ( $x, y$ )
- The letter  $b$  represents the \_\_\_\_\_, this is where the line crosses the horizontal axis.
- **Q: What are the restrictions on this value?**  
**A:** \_\_\_\_\_.
- **Example: Below are three different graphs along with 4 different equations. Match them.**

Equations: 1.  $y = -0.5x$    2.  $y = 3$    3.  $x = -3$    4.  $y = 2x + 5$



Polynomials:

- Recall: a \_\_\_\_\_ is an expression of the form:

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

where the  $a_i$ 's are real number \_\_\_\_\_ .

- For nonzero  $a_n$ , the expression is said to be of  $n$ th degree (the highest power is  $n$ ), the \_\_\_\_\_ is  $a_n x^n$  and the \_\_\_\_\_ is  $a_n$ .

- Examples of polynomials that are common

Degree	Name	Form
0	Constant	$f(x) = c$
1	Linear	$f(x) = mx + b$
2	Quadratic	$f(x) = ax^2 + bx + c$
3	Cubic	$f(x) = ax^3 + bx^2 + cx + d$

Power Functions:

- A \_\_\_\_\_ is of the form  $f(x) = x^a$ , where  $a$  is any constant.

- Q: What is the difference between a power function and a polynomial?**

**A:** \_\_\_\_\_  
\_\_\_\_\_

- Q: Is a polynomial a power function?**

**A:** \_\_\_\_\_  
\_\_\_\_\_

- If the value of  $a$  is a fraction, the power function is also called a \_\_\_\_\_ .

- Example.*  $f(x) = x^{1/2} = \sqrt{x}$

- If the value of  $a$  is negative, it is the \_\_\_\_\_ .

- Example.*  $f(x) = x^{-1} = \frac{1}{x}$

Rational Functions:

- A rational function is a ratio of two polynomials  $\frac{p(x)}{q(x)}$ .

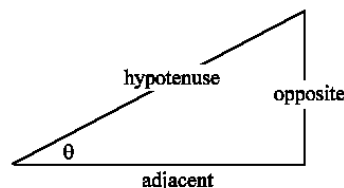
- If the rational function has a root, it would be when  $p(c) = 0$  (so long as  $q(c)$  isn't zero).

- The rational function will be undefined (at  $c$ ) whenever  $q(c) = 0$ .

Trigonometric Functions:

- The six trig functions are defined in terms of right triangles (see figure). The two “most important” being sine and cosine.

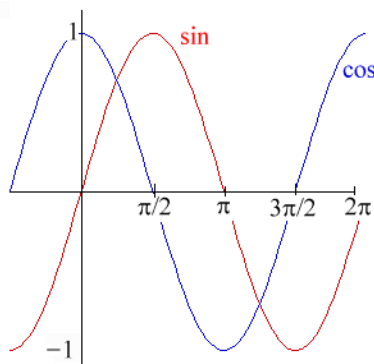
$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$



- All other trig functions can be defined in terms of sine and cosine, so remember the definitions above, and the relationships between them and the others...

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

- To evaluate trig functions, remember some fundamentals.
- First, remember the basic shape of  $\sin(x)$  and  $\cos(x)$ , along with when they are 0, and  $\pm 1$ .



- From the figure you can see they are periodic, both with a period of  $2\pi$ .

Q: When is sine equal to zero?

A: \_\_\_\_\_.

Q: When is cosine equal to zero?

A: \_\_\_\_\_.

Exponential Functions:

- $y = kb^t$
- Variables are  $y$  and  $t$ .  $y$  is the dependent variable and  $t$  is the independent variable.
- $b$  is the \_\_\_\_\_ ( $b > 0$ ,  $b \neq 1$ ).
- $k$  is the \_\_\_\_\_ (when  $t = 0$ ).
- A commonly used base is  $e = 2.7182818284\dots$
- *Rules for Exponentials:*

$$b^x b^y = b^{x+y}$$

$$(b^x)^y = b^{xy}$$

$$b^{-x} = \frac{1}{b^x}$$

Log Functions:

- We want to undo the exponential function  $b^y = x$
- This is true if and only if  $y = \log_b x$   
 $y$  is the exponent,  $b$  is the base, and  $x$  is the argument.
- So the log function is the \_\_\_\_\_ of the exponential function.

- **Q:** What are our conditions on  $x$ , and  $b$ ?

**A:** \_\_\_\_\_.

- *Example. Change to exponential form to solve  $\log_{10} \frac{1}{100} = ?$*

- *Example. Convert  $e^{-t} = 4000$  to log.*

- **Special Log Bases:**

\_\_\_\_\_ is natural log (written  $\ln$ )  
\_\_\_\_\_ is common log (written  $\log$ )

- These will be the only two on your calculator. So if you need to calculate say,  $\log_4 2$ , you have to use the change of base formula  $\log_b M = \frac{\log_a M}{\log_a b}$ .

- *Example, use the change of base formula to evaluate  $\log_4 2$*

### Algebraic and Transcendental Functions

- An \_\_\_\_\_ is constructed with algebraic operations (addition, subtraction, multiplication and division). A \_\_\_\_\_ is anything else.
- **Q: Classify all the functions we have looked at so far as algebraic or transcendental... Log, Exponential, Trig, Rational, Power, Polynomials**
- **A: \_\_\_\_\_.**

### Transformations of Functions

- NOTE: There are ways of transforming functions by shifting them left, right, up or down. Or by stretching them, or shrinking them. Or by reflecting them... BUT... once we learn the *actual* tools for graphing (after differentiation) it usually more straightforward to graph what you are given.
- So we will not go over transformations in class, but I expect you will look over the material and bring me any questions you might have.
- Some functions (like trigs, exponentials and logs) are, admittedly, easier to do with transformations.

### Composition of Functions:

- Sometimes it is helpful to break things into different parts, and recognize how they fit together.
- A composite function works to accomplish this goal, because it identifies functions inside functions.
- For example, we can think of the function  $f(x) = (x+1)^2$  as the composite of two functions, one function is the inside piece  $in(x) = x+1$  and the other is the outside piece  $out(x) = x^2$ . So we see that  $f(x) = out(in(x))$ .
- The \_\_\_\_\_  $f \circ g(x)$ , the **composition** of  $f$  and  $g$ , is defined as  $(f \circ g)(x) = f(g(x))$ . The domain of  $g$  is  $x$  and the domain of  $f$  is  $g(x)$ .
- *Example. Find  $(g \circ h)(1/2)$  if  $g(x) = x^2 - 2x - 6$  and  $h(x) = x^3$*

- *Example.* Find the composites and domains if  $f(x) = \frac{6}{x}$  and  $g(x) = \frac{1}{2x+1}$

