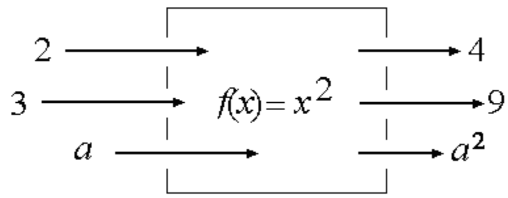
Section 1.1 – Functions and Their Representations

Basic Definitions:

- A **function $f(x)$** is a rule that assigns to each input x exactly one output f



- For the example given above, $f(x) = x^2$, $f(1) = 1$ and $f(-1) = 1$.
- Q: Since $f(1) = f(-1)$ does this mean that it is *not* a function?
  A: No, because the inputs (1 and –1) have just one output, it is ok if they are the same.
- The set of all inputs is collectively called the **domain**, and the set of all outputs is collectively called the **range**.
- Q: For the function $f(x)$, which is the dependent variable and which is the independent variable?
  A: The dependent variable is f and the independent variable is x.

Domain:

- The domain of a function can be given or implied.
- *Example of given domain:* $f(x) = x^2$, $x > 0$.

  The domain is given to you with the text $x > 0$.
  It means for any other values of x ($x \le 0$) the function is not defined.

- *Example of implied domain:* $f(x) = \sqrt{x}$.

  This domain is implied because the square root function is not defined for negative values of x.
  This means that the domain of f given above is $x \ge 0$, even though we are not directly told this.

- How do we find the domain if it is not given? It has to do with when your function is defined. Find the values when your function is undefined, and the domain is all others.
- *Example: What is the domain of the functions* $f(x) = \sqrt{1-x}$, $g(x) = \dfrac{1}{x+2}$ and $h(x) = \log(x)$?

  $f(x)$: $1 - x \ge 0 \Rightarrow x \le 1$.

  $g(x)$: $x \ne 2$.

  $h(x)$: $x > 0$

Representing a Function:

We can represent a function the following ways:

- **Algebraically** with a formula
  i.e. $f(x) = x^2$

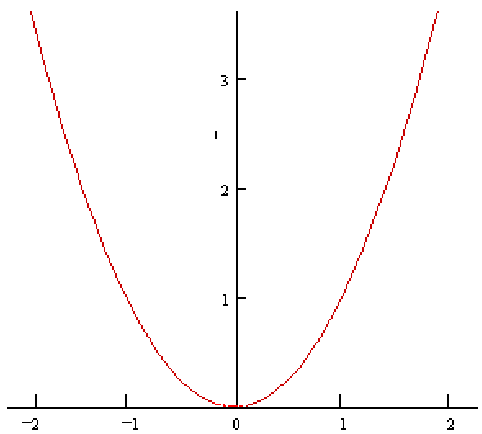
- **Verbally** by describing what it does in words
  i.e. Squaring the original value.

- **Numerically** by a table of values with inputs and matched outputs

x	y
-1	1
0	0
1	1
2	4

NOTE: The inputs chosen for the table can be anything that is in the domain

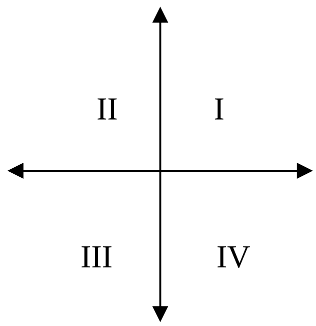
- **Visually** with a graph



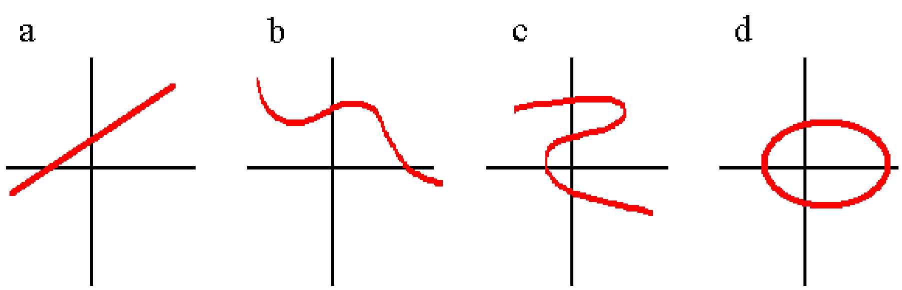
Some Graphing Standards:

- The vertical and horizontal lines are known as **axes**. The **vertical axis** and the **horizontal axis**. They are simply number lines that cross each other at zero.
- NOTE: Many refer to them as the *x* axis and the *y* axis, but this is a bad idea because what if you don't have an *x* and a *y*, but a *g* and a *t*?
- All points on this two dimensional plane are called ordered pairs (*x*,*y*) where *x* represents the horizontal distance (it is the independent variable) and *y* represents the vertical distance (it is the dependent variable).
- Q: Why do we refer to this as a two dimensional plane?
  A: Because we have two ways to move, left-right and up-down.
- The ordered pair (0,0) is known as the **origin**.
- The negative values for the horizontal axis are always to the left of the origin, and the negative values for the vertical axis are below the origin.

- The plane is divided into 4 different regions, called **quadrants**, and they are labeled in a particular way…



- So let's see if we have an understanding of some of the basics…
  - Q: Where is the origin in the above graph?
    A: Where the lines cross
  - Q: What sign will the first coordinate have in Quadrant I?
    A: Positive (in fact, both coordinates are positive in Quadrant I).
  - Q: What sign will the second coordinate have in Quadrant IV?
    A: In Quadrant IV, the first coordinate is positive (the second negative).
  - Q: What Quadrant will the point (0.5, –2) be in?
    A: From the origin, you would move 0.5 right, then down 2, so Quadrant IV.
- A **function *y*(*x*)** is a rule that assigns to each input *x* exactly one output *y*.
- Q: So every *x* has only one *y*. Which of the following graphs are functions and which are not?



A: a and b are functions, c and d are not.

- The **vertical line test** is a way to test whether a graph is a function. Any vertical line cannot pass through a function more than once.
- Q: How does the vertical line test fit with the definition of a function?
  A: Since each *x* can have only one *y*, then a vertical line passed through will only hit one *y* (not more than one, or some *x* would have more than one *y*).

Piecewise Functions:

- When the domain is broken up into pieces, and different functions apply to different parts of the domain, the function is known as a **piecewise function**.
- The most popular (but often misconstrued) piecewise function is the absolute value function. The definition of the absolute value function is given as:

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x \leq 0 \end{cases}$$

  It is often taught in school as "strip away the minus if it's positive," but this is actually incorrect. If the value of x is negative, you multiply x by a negative 1.

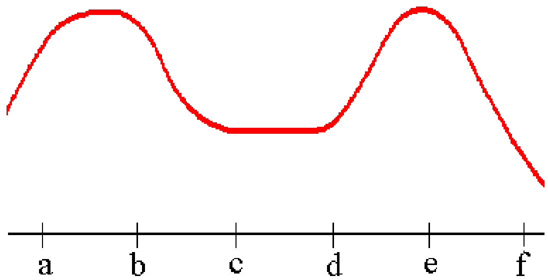
- Trouble may occur when you move away from numbers into symbols.
  Q: What is the absolute value of a?
  A: Well that depends, if a is negative, then the absolute value of a is $-a$
- *Example. For the function* $f(x) = \begin{cases} -5x-8 & x < -2 \\ \frac{1}{2}x+5 & -2 \leq x \leq 4 \\ 10-2x & x > 4 \end{cases}$ *find*

  $f(-4) = -5(-4) - 8 = 12$
  $f(-2) = \frac{1}{2}(-2) + 5 = 4$
  $f(4) = \frac{1}{2}(4) + 5 = 7$
  $f(6) = 10 - 2(6) = -2$

Increasing and Decreasing Functions:

- Increasing/decreasing is a way to describe what happens when you compare successive values of $f(x)$. You always move from left to right.
- If a function rises from left to right (the dependent variable goes up as the independent variable moves right) the function is said to be **increasing** over that domain.
- If a function drops from left to right (the dependent variable goes down as the independent variable moves right) the function is said to be **decreasing** over that domain.
- If a function stays the same from left to right (the dependent variable stays the same as the independent variable moves right) the function is said to be **constant** over that domain.

- *Example: On the domains given, is the following function increasing, decreasing, constant or neither?* (a,b) (b,c) (c,d) (d,e) (e,f) (a,c) (c,e) (b,e)

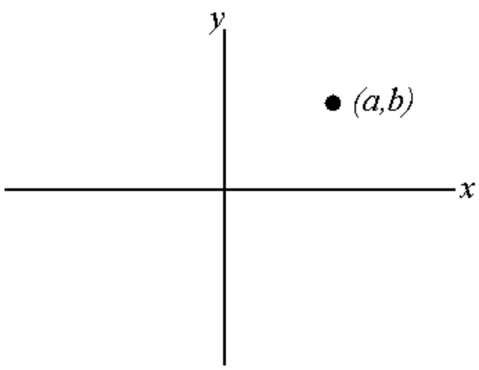


(a,b)-neither (b,c)-dec (c,d)-const (d,e)-inc (e,f)-dec
(a,c)-neither (c,e)-neither (b,e)-neither

- Formally, a function is **increasing** on the open interval I if for all a and b in I with $a < b$, $f(a) < f(b)$.
- Formally, a function is **decreasing** on the open interval I if for all a and b in I with $a < b$, $f(a) > f(b)$.
- Formally, a function is **constant** on the open interval I if for all a and b in I with $a < b$, $f(a) = f(b)$.
- NOTE: Increasing and decreasing mean strictly, there is no equality allowed. Also, we only talk about increasing/decreasing on open intervals (not closed intervals).

Symmetry:

- Being able to see the symmetry in functions helps to analyze their behavior



- *Example: We can move a point (a,b) located in Quadrant I to any other quadrant by reflecting it…*
  (a,b) reflected in the x axis will result in the point $(a,-b)$
  (a,b) reflected in the y axis will result in the point $(-a,b)$
  (a,b) reflected through the *origin* will result in the point $(-a,-b)$.
- We define symmetry based on these reflections. If any point (x,y) on a graph has a corresponding point $(x,-y)$ then the graph is said to be **symmetric with respect to the x axis**. In other words, above and below the x axis are mirror images of each other.

- If any point (x,y) on a graph has a corresponding point (–x,y) then the graph is said to be **symmetric with respect to the *y* axis**. In other words, the left and right of the y axis are mirror images of each other.
- If a function, f, is symmetric with respect to the y axis, it is considered to be an **even function**. It can be tested by the fact that for every x in the domain of f, $f(x) = f(-x)$.
- Q: How does 'even' relate to the definition of symmetric with respect to the y axis?
  A: If $f(x) = f(-x)$, then the corresponding points are (x, f) and (–x, f) which fits the definition.
- If any point (x,y) on a graph has a corresponding point (–x,–y) then the graph is said to be **symmetric with respect to the origin**. In other words, the graph is the same if we rotate it 180 degrees (turn it upside down).
- If a function, f, is symmetric with respect to the *origin*, it is considered to be an **odd function**. It can be tested by the fact that for every x in the domain of f, $f(-x) = -f(x)$.
- Q: How does 'odd' relate to the definition of symmetric with respect to the origin?
  A: If $f(-x) = -f(x)$, then the corresponding points are (–x, f) and (– (–x), – f) which fits the definition.
- To determine if a function is even or odd (without graphing), analyze $f(-x)$. It need not be even or odd (it may be neither).
- *Example. Determine even/odd for* $f(x) = x + \frac{1}{x}$

$$\begin{aligned} f(-x) &= -x - \frac{1}{x} \\ &= -\left(x + \frac{1}{x}\right). \\ &= -f(x) \end{aligned}$$

So f is odd.