How to Sketch the Graph of a Function $f(x)$:

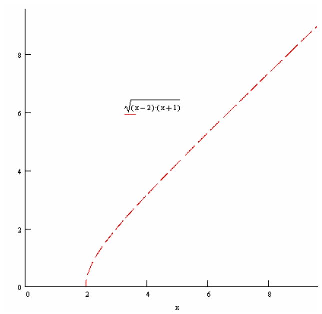
(types we have seen so far)

Identify the function type

1. **Algebraic**

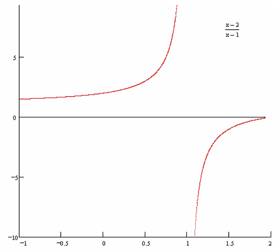
Root Functions $f(x) = \sqrt[a]{g(x)}$

- Find the domain: If a is even then $g(x) \geq 0$ is the domain
- Find the roots: when $g(x) = 0$
- Analyze the first and second derivatives to determine the shape[1]
- Sketch using the critical points and intercepts



Rational expressions $f(x) = \dfrac{p(x)}{q(x)}$

- Find the domain. The point(s) $x = c$ is a domain restriction when $q(c) = 0$. If $p(c) \neq 0$ then c is a vertical asymptote
- Find the roots: The point(s) $x = c$ is a root if $p(c) = 0$ AND $q(c) \neq 0$
- Find horizontal asymptotes (if any): Take the limit as x tends to positive and negative infinity by looking at the leading terms of p and q
- Sketch the asymptotes and roots
- Analyze the first and second derivatives to determine the shape[1]
- Sketch using the asymptotes, critical points, IPs and intercepts



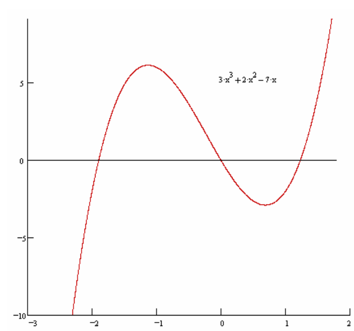
Polynomials (domain is all real x values)

Linear $f(x) = mx + b$:
- b is the y-intercept
- m is the slope of the line (rise / run)

Quadratic $f(x) = ax^2 + bx + c$:
- The axis of symmetry is $-b/2a$
- The discriminant $b^2 - 4ac$ gives the number of x-intercepts
- The sign of a determines whether it opens up or down

Other Polynomials

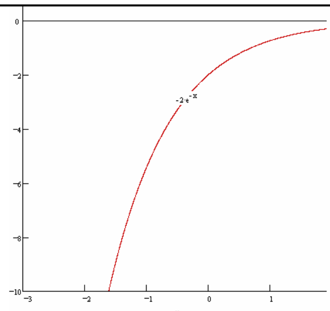
$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$
- The leading term corresponds to the highest power of x
- Using the leading term, evaluate the limit at positive and negative infinity
- If the polynomial factors, find the roots
- Analyze the first and second derivatives to determine the shape[1]
- Sketch using the critical points, IPs, intercepts



2. **Transcendental**

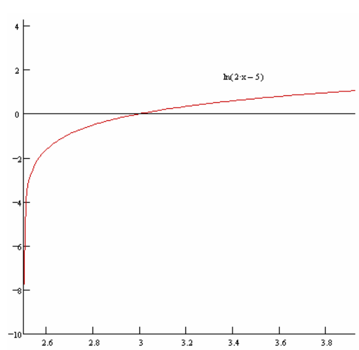
Exponentials $f(x) = k \cdot b^x$

- $b > 1$ implies growth
  $b < 1$ implies decay
- If $k > 0$, the function will always be positive. As x tends to infinity, it will tend to 0 (for decay) and infinity (for growth)
- If $k < 0$, the function will always be negative (it is flipped upside down)
- The y intercept is k
- Shifts:
  $f(x) = kb^x + c$, shift up ($c > 0$) or down ($c < 0$) by c
  $f(x) = kb^{x+c}$, shift left ($c < 0$) or right ($c > 0$) by c

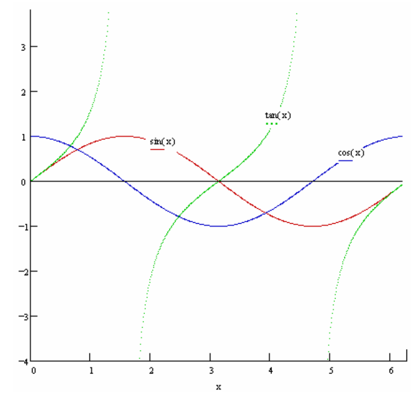


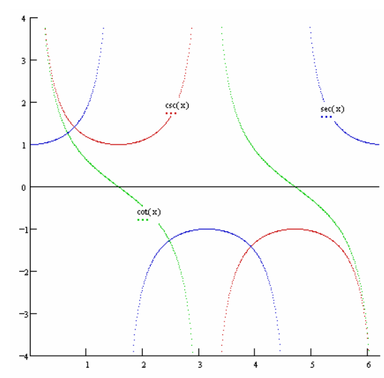
Logs $f(x) = \ln[g(x)]$

- This is log base e
- The domain is restricted to $g(x) > 0$
- It has a vertical asymptote at $g(x) = 0$
- $\ln[g(x)]$ tends to infinity as $g(x)$ tends to infinity

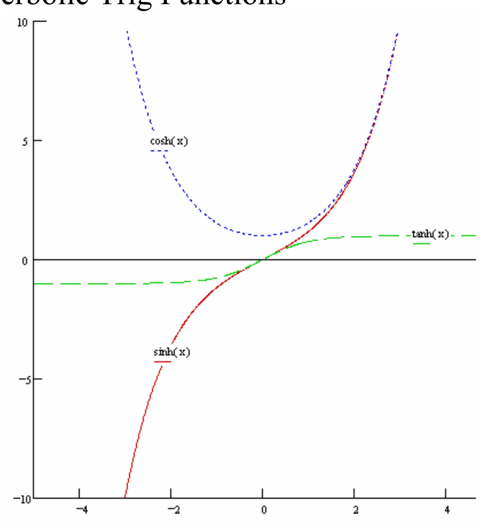


Trig Functions





Hyperbolic Trig Functions



How to Sketch the Graph of a Function $f(x)$:

[1]Analyze the First and Second Derivatives to Determine Shape

- Find $f'(x)$
- Find critical points (CP) – wherever $f'(x) = 0$ or undefined
- Find $f''(x)$
- Find inflection points (IP) – wherever $f''(x) = 0$
- Make sign diagram for $f'(x)$ and $f''(x)$ which contains all CP's, IP's (and vertical asymptotes, if there are any)
- Below the sign diagram, sketch the "shape" of the graph (i.e. increasing "/", decreasing "\", horizontal "–" etc.)
- Find the actual critical points by finding $f(CP), f(IP)$, etc.
- Plot CP's and IP's, and x and y intercepts.
- Sketch