Sta 731 - Homework 4

Due Tuesday, 10/9

1. IF X is a random variable, so is $|X|$.

2. IF $F(x) = P(X \leq x)$ is continuous in x, show that $Y = F(X)$ is measurable and that Y has a uniform distribution $P(Y \leq y) = y$, $0 \leq y \leq 1$

3. Let $(\Omega, \mathcal{B}, P)$ be a probability space with $\mathcal{B} = \sigma(\mathcal{A})$, where $\mathcal{A}$ is an algebra of subsets of Ω. Show that for $B \in \mathcal{B}$ and all $\epsilon > 0$, there exists a set $A_\epsilon \in \mathcal{A}$ such that $P(B \Delta A_\epsilon) < \epsilon$ and so $|P(B) - P(A_\epsilon)| < \epsilon$.

4. Let $\Omega = [0, 1]$, $\mathcal{B}$=Borel sets, P=Lebesque measure. Show that $(\Omega, \mathcal{B}, P)$ is a universal probability space in the sense that if F is any proper distribution function on $\mathbb{R}$ ("proper" means that $F(\infty) = 1$, $F(-\infty) = 0$), there is a random variable X on $(\Omega, \mathcal{B}, P)$ with distribution function F.

   Hint: Define $G(y) = \sup\{x \in \mathbb{R} : F(x) < y\}$, $0 < y < 1$, and take $X(\omega) = G(\omega)$ with $X(0)$ and $X(1)$ arbitrary.