Sta 731 - Homework 3

Due Thursday, 9/27

1. Page 36: Show that the *new* λ-system postulates $(\lambda_1, \lambda_2, \lambda_3)$ implies the *old* postulates $(\lambda_1', \lambda_2', \lambda_3')$.

2. Problem 1, Exercise 2.6, Page 63: Let Ω be a non-empty set. Let $\mathcal{F}_0$ be the collection of all subsets such that either A or A^c is finite.
(a) Show that $\mathcal{F}_0$ is a field.
Define for $E \in \mathcal{F}_0$ the set function P by

$$P(E) = \begin{cases} 0, & \text{if } E \text{ is finite} \\ 1, & \text{if } E^c \text{ is finite.} \end{cases}$$

(b) If Ω is countably infinte, show P is finitely additive but not σ-additive.
(c) If Ω is uncountable, show P is σ-additive on $\mathcal{F}_0$.

3. Problem 3, Exercise 2.6, Page 63: Let$(\Omega, \mathcal{B}, P)$ be a probability space. Show for events $B_i \subset A_i$ the following generalization of subadditivity:

$$P(\bigcup_i A_i) - P(\bigcup_i B_i) \leq \sum_i (P(A_i) - P(B_i)).$$

4. Let μ be a nonnegative, finitely additive set function on the field $\mathcal{F}$. If $A_1, A_2, ...$ are disjoint sets in $\mathcal{F}$ and $\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$, show that

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) \geq \sum_{n=1}^{\infty} \mu(A_n)$$