Sta 731 - Homework 2

Due Tuesday, 9/18

1. Problem 12, Exercise 1.9, Page 22: Let $\Omega = \{1, 2, 3, 4, 5, 6\}$ and let $\mathcal{C} = \{\{2, 4\}, \{6\}\}$. What is the field generated by $\mathcal{C}$ and what is the σ- field?

2. Problem 20 (with minor correction), Exercise 1.9, Page 23: Suppose $\mathcal{C}$ is a non-empty class of subsets of Ω. Let $\mathcal{A}(\mathcal{C})$ be the minimal field over $\mathcal{C}$. Show that $\mathcal{A}(\mathcal{C})$ consists of sets of the form
$$\bigcup_{i=1}^{m} \bigcap_{j=1}^{n_i} A_{ij},$$
where for each i,j either $A_{ij} \in \mathcal{C}$ or $A_{ij}^c \in \mathcal{C}$. Thus, we can explicitly represent the sets in $\mathcal{A}(\mathcal{C})$ even though this is impossible for the σ- field over $\mathcal{C}$.

3. Problem 21, Exercise 1.9, Page 23: Suppose $\mathcal{A}$ is a field and suppose also that $\mathcal{A}$ has the property that it is closed under countable disjoint unions. Show $\mathcal{A}$ is a σ-field.

4. Problem 28, Exercise 1.9, Page 24: Show that the periodic sets of $\mathbb{R}$ form a σ- field; that is, let $\mathcal{B}$ be the class of sets A with he property that $x \in A$ implies $x \pm n \in A$ for all natural numbers n. Then show $\mathcal{B}$ is a σ- field.

5. Problem 30, Exercise 1.9, Page 24: Let $\mathcal{B}_i$ be σ- fields of subsets of Ω for $i = 1, 2$. Show that the σ- field $\mathcal{B}_1 \bigvee \mathcal{B}_2$ defined to be the smallest σ- field containing both $\mathcal{B}_1$ and $\mathcal{B}_1$ is generated by sets of the form $B_1 \bigcap B_2$ where $B_i \in \mathcal{B}_i$ for $i = 1, 2$.